

Reticulated pattern formation during folding instabilities in thin films: what does it teach us about venation?



Manouk Abkarian



Collaborators:

Pilnam Kim

Howard A. Stone



1

Instabilities in compression

Instabilities limit loading of layered materials: delamination

Wear resistant coating



Optical coating

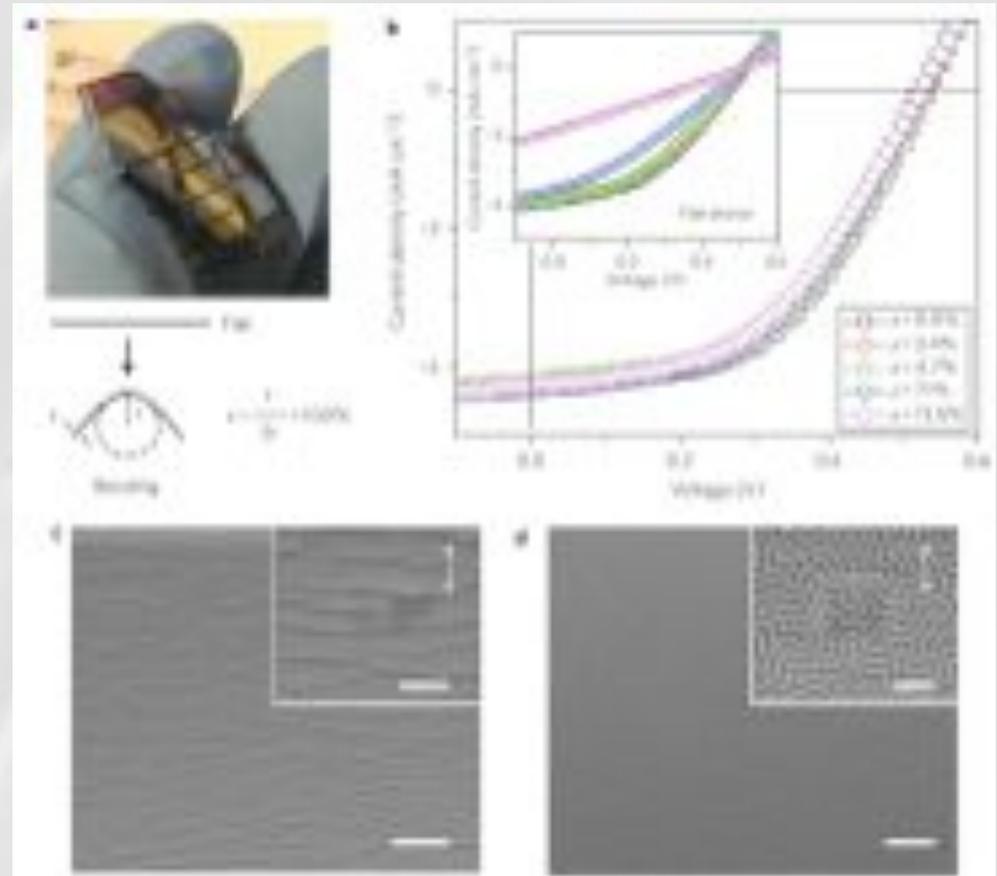
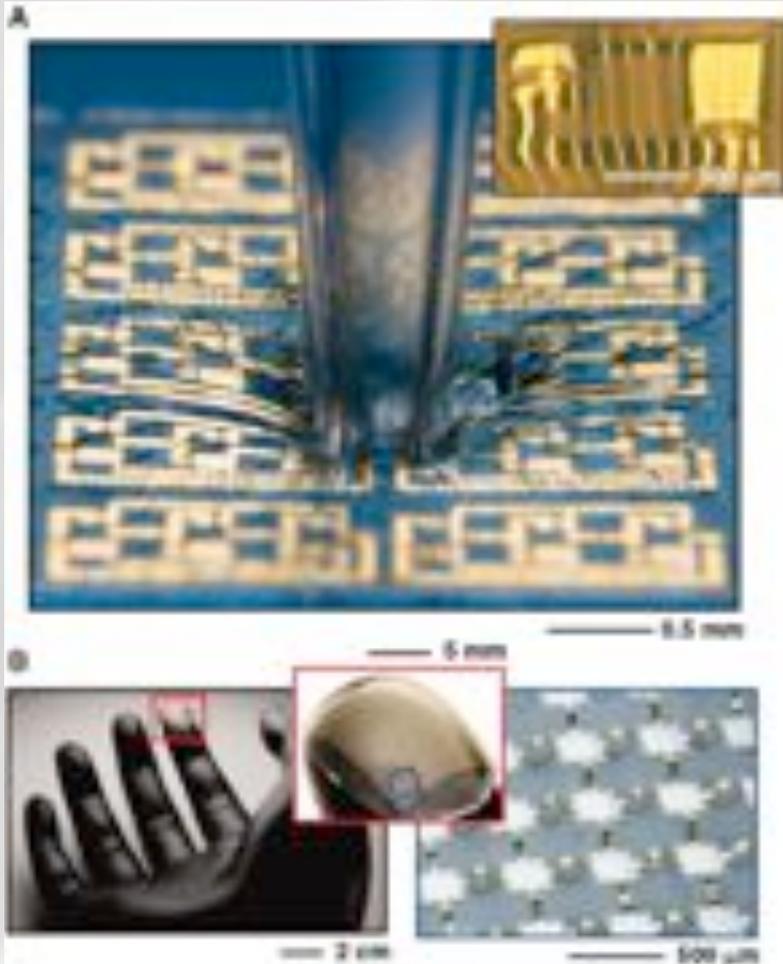


Instabilities in compression

Whose understanding and control finds original applications :

Flexible electronic devices

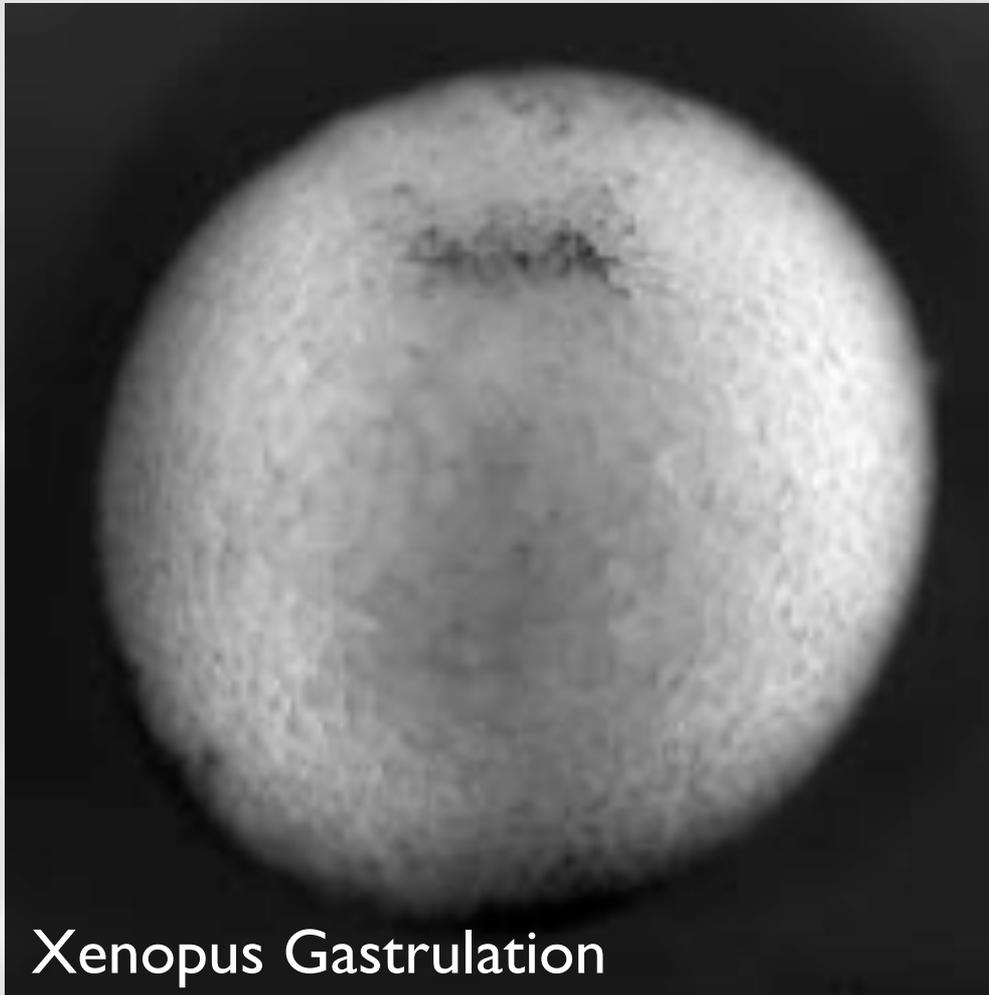
Resilient Light harvesting films



Kim et al. Nature Photonics,

Instabilities in compression

And may be understand some aspects of morphogenesis ?



Folding ?

Creasing ?

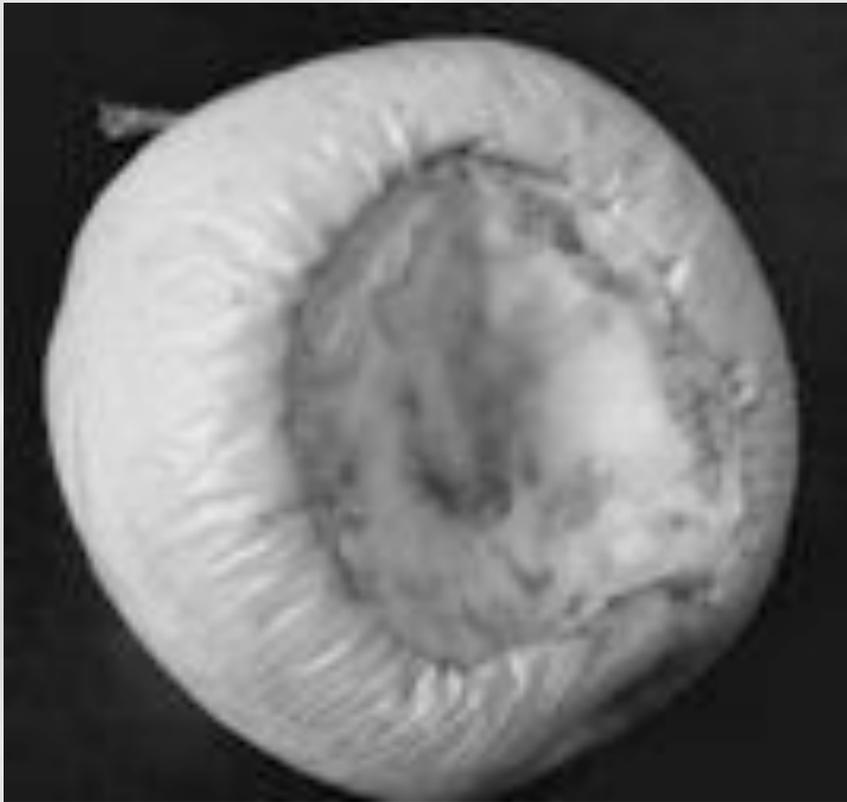
Crumpling ?

... ????

<http://www.gastrulation.org>

Instabilities in compression

Drying fruits ?



Aging Skin ?



Euler Buckling of a free-standing film



$$E = h^3 \times (\text{bending}) + h \times (\text{stretching})$$

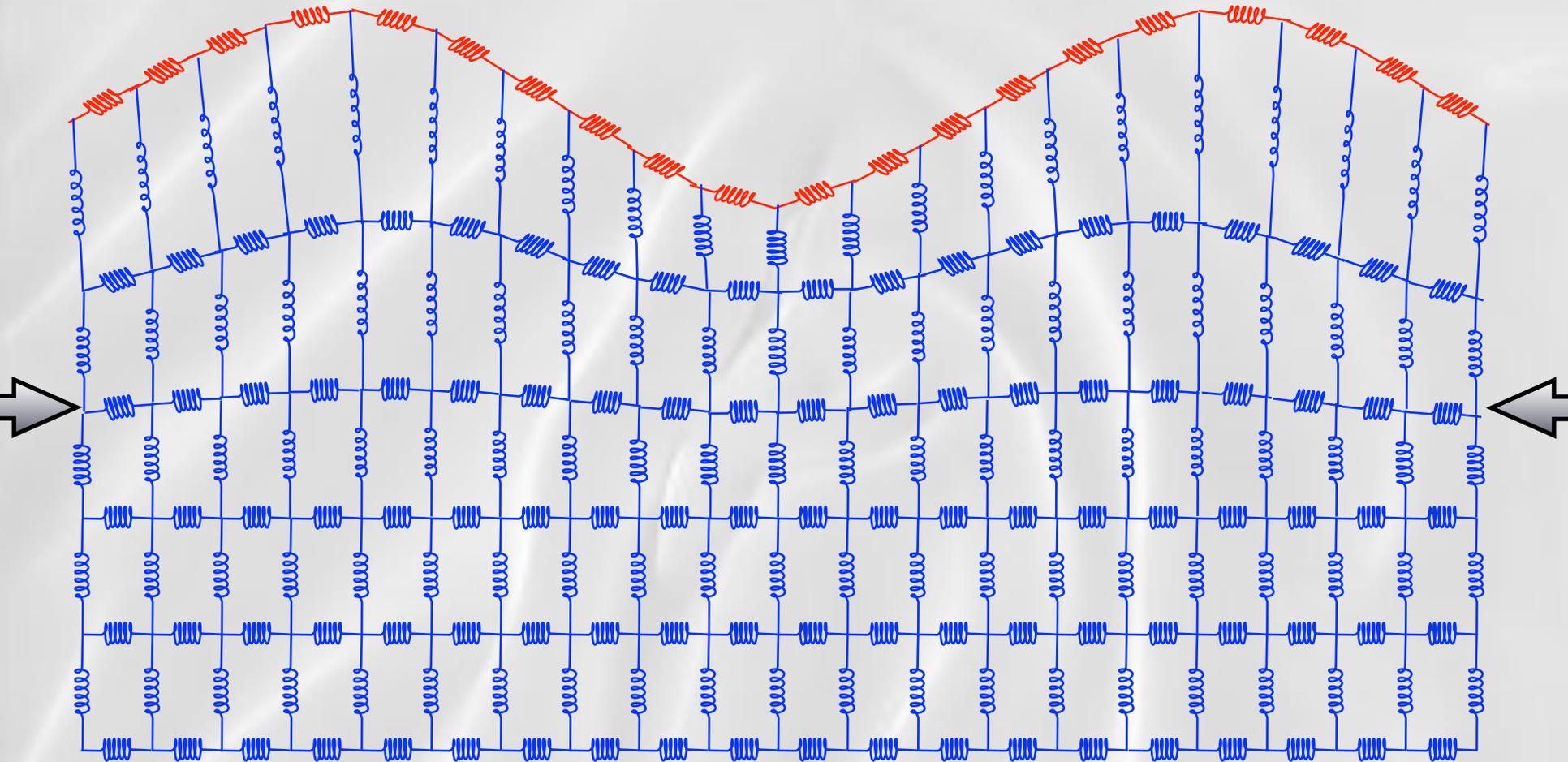
$h \Rightarrow 0$, bending is preferred by the stripe

Presence of a foundation

$$E = \boxed{B} \times (\text{film bending}) + \boxed{K} \times (\text{substrate stretching})$$

B: bending stiffness

K: 'effective' stiffness

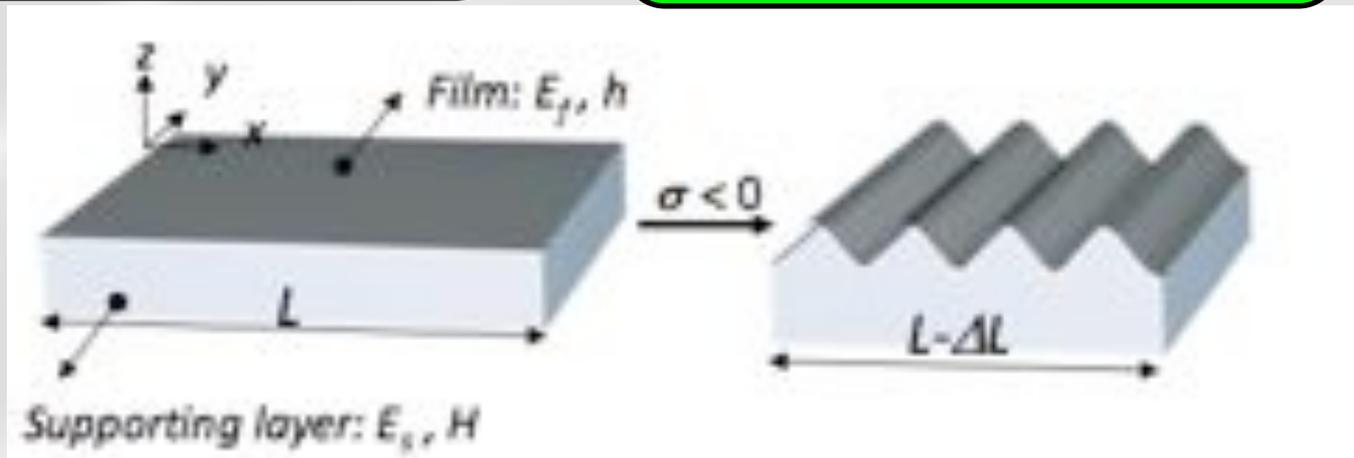


Presence of a foundation

$$E = B \times (\text{film bending}) + K \times (\text{substrate stretching})$$

B: bending stiffness

K: 'effective' stiffness



$$h \ll H$$

$$B \sim E_f \times h^3$$

Elastic foundation $K = K(E_s, \sigma) \approx E_s / \lambda$

Liquid foundation $K \approx \rho g$

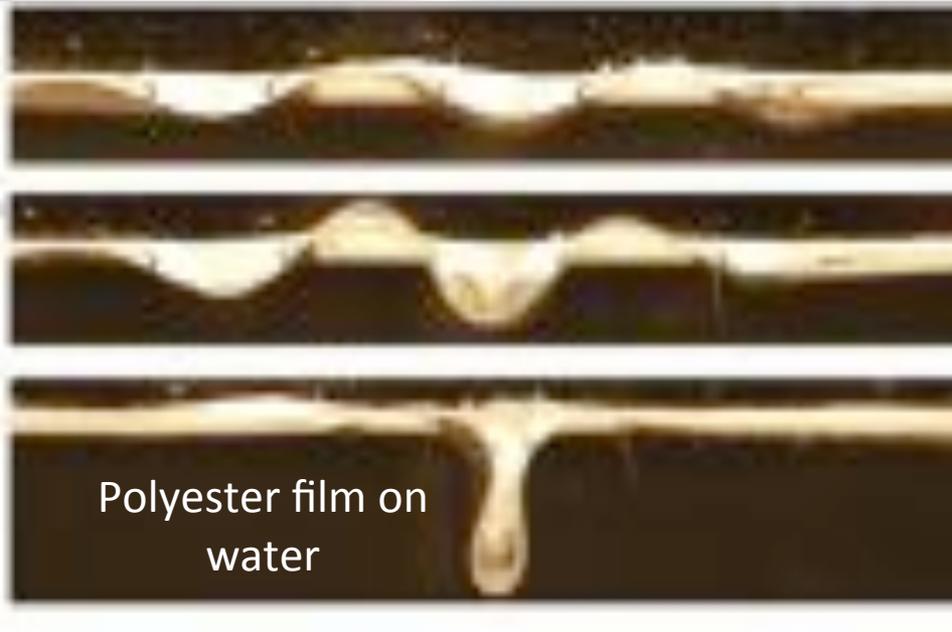
$$\varepsilon_{c,w} \sim \left(\frac{E_s}{E_f} \right)^{2/3}, \quad \lambda \sim \left(\frac{B}{K} \right)^{1/4}, \quad A \sim \varepsilon^{1/2} \lambda$$

Evans et al, Nature 1998
Cerde et al, PRL 2003

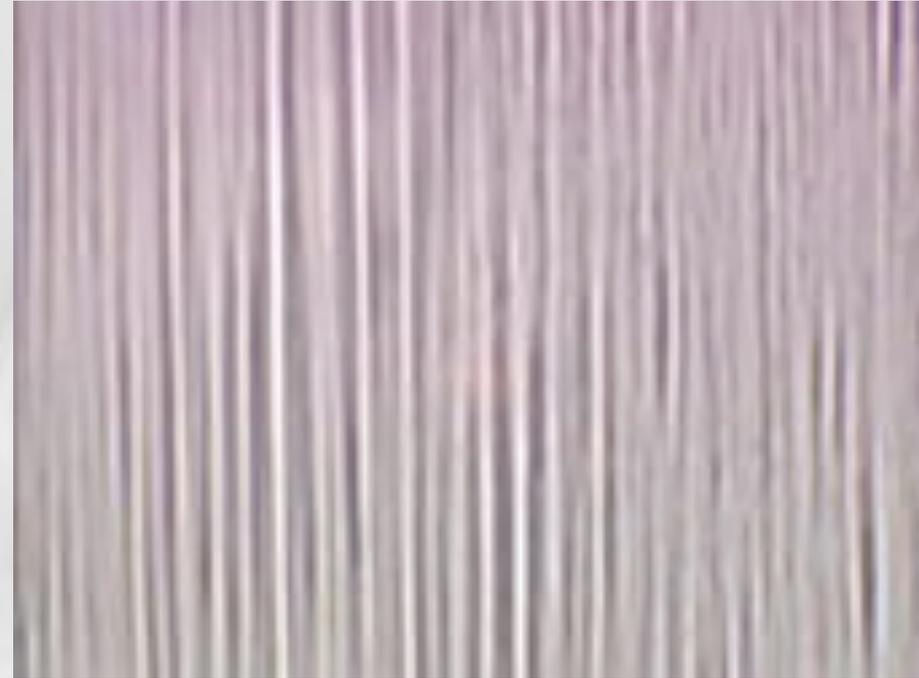
Wrinkles are unstable

Uniaxial stress: on Liquid foundation

Pocivavsek *et al*, *Science* (2008)



Leahy *et al*, *PRL* (2010)



Gold nanoparticles film

Wrinkles are unstable

Uniaxial stress: On elastic substrate



Brau et al. Nature Physics, 2010

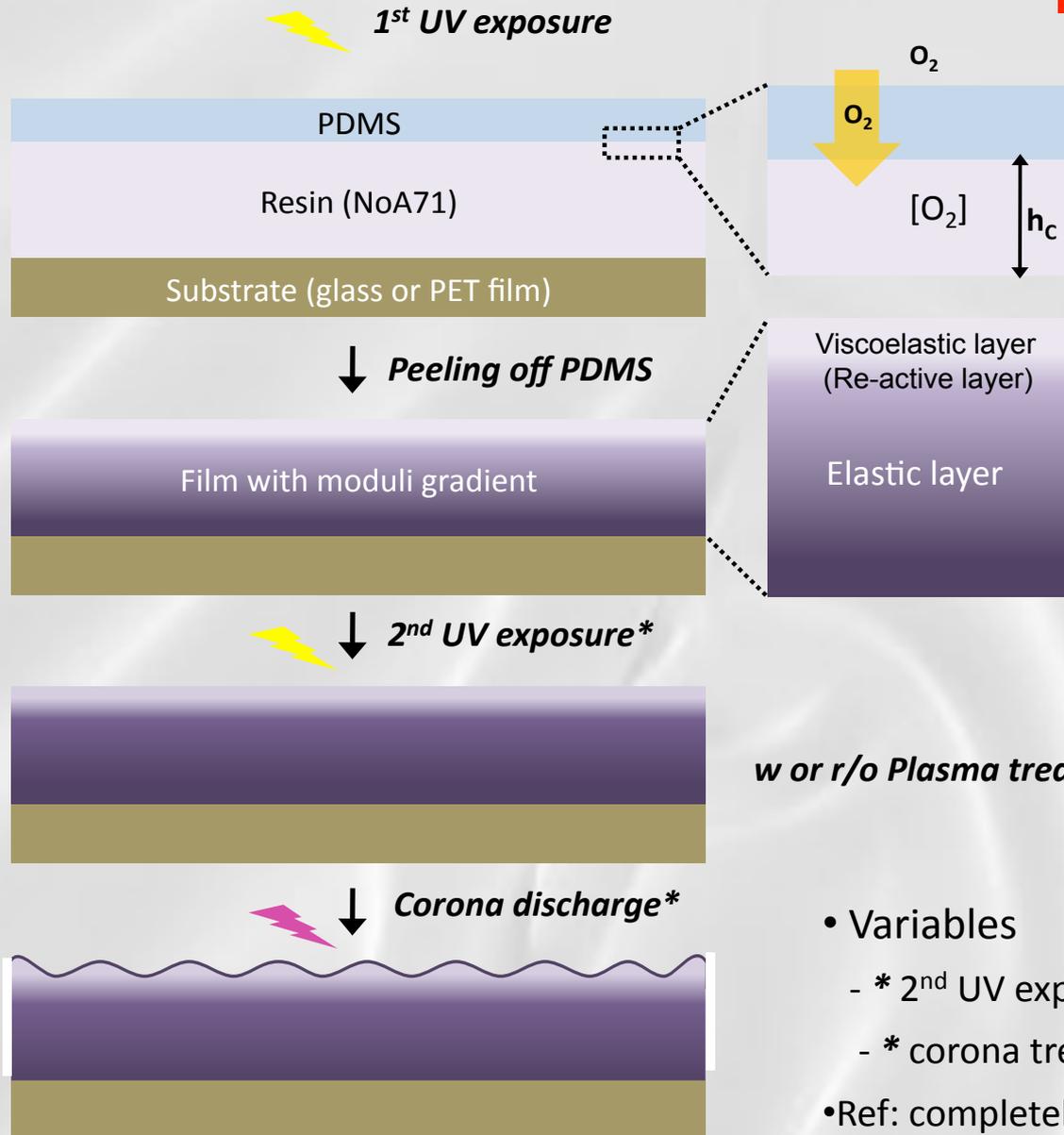
Biaxial case: our questions

What happens in the more general biaxial case ?

Canonical example of the paper ball

What is the dynamics of proliferation of these localizations ?

How to make a crust in compression ?



*w or r/o Plasma treatment***

- Variables

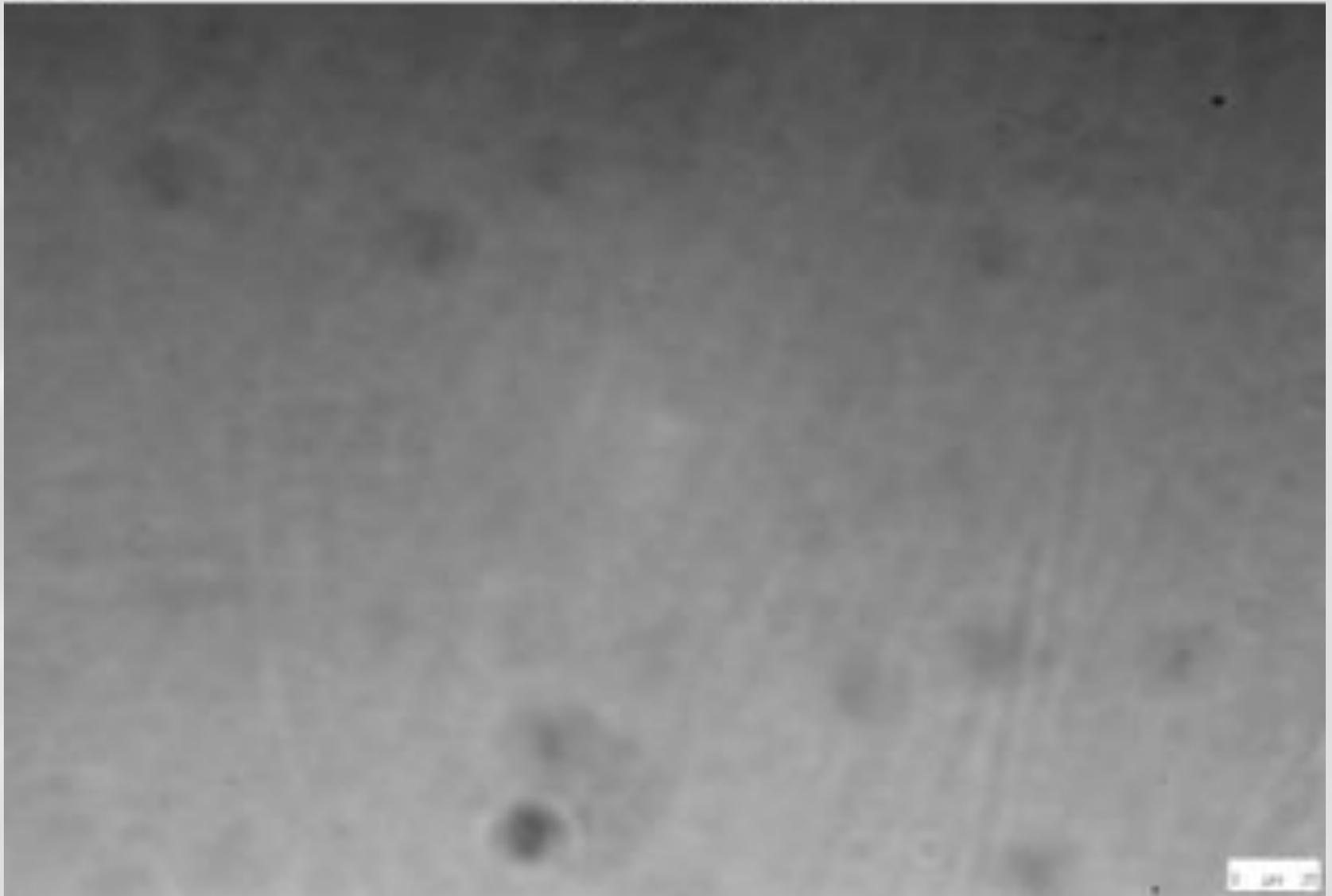
- * 2nd UV exposure : control of E_s/E_f
- * corona treatment: increasing strain

- Ref: completely cured NoA ~ 400 MPa

Wrinkling instability

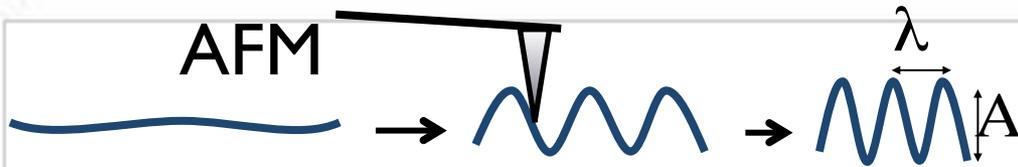
0 sec

Wrinkling instability

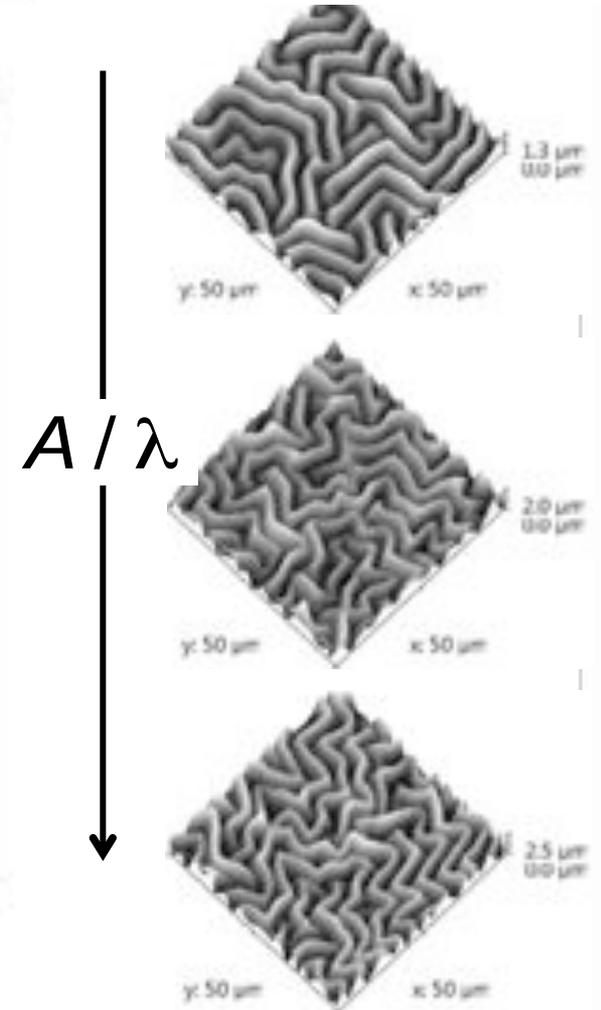
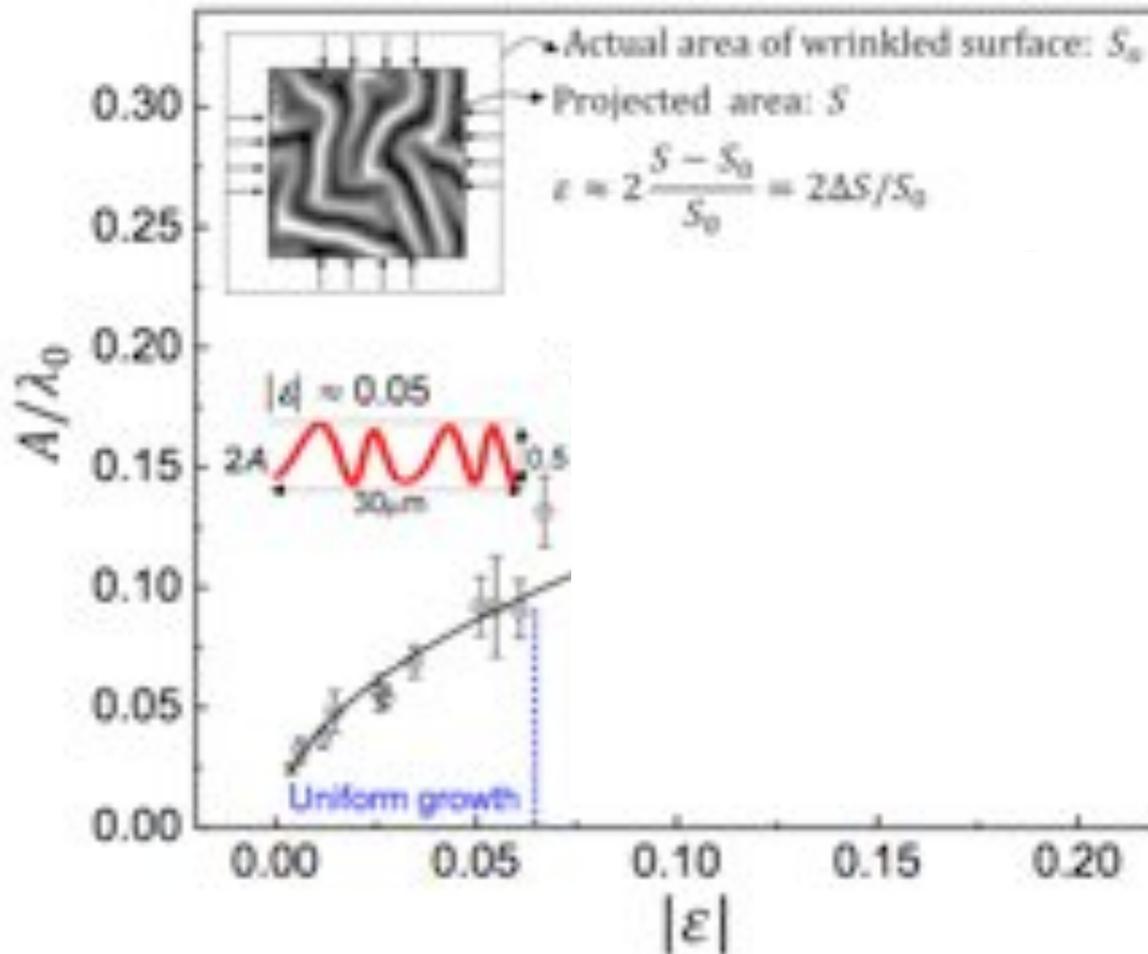


Wrinkling instability

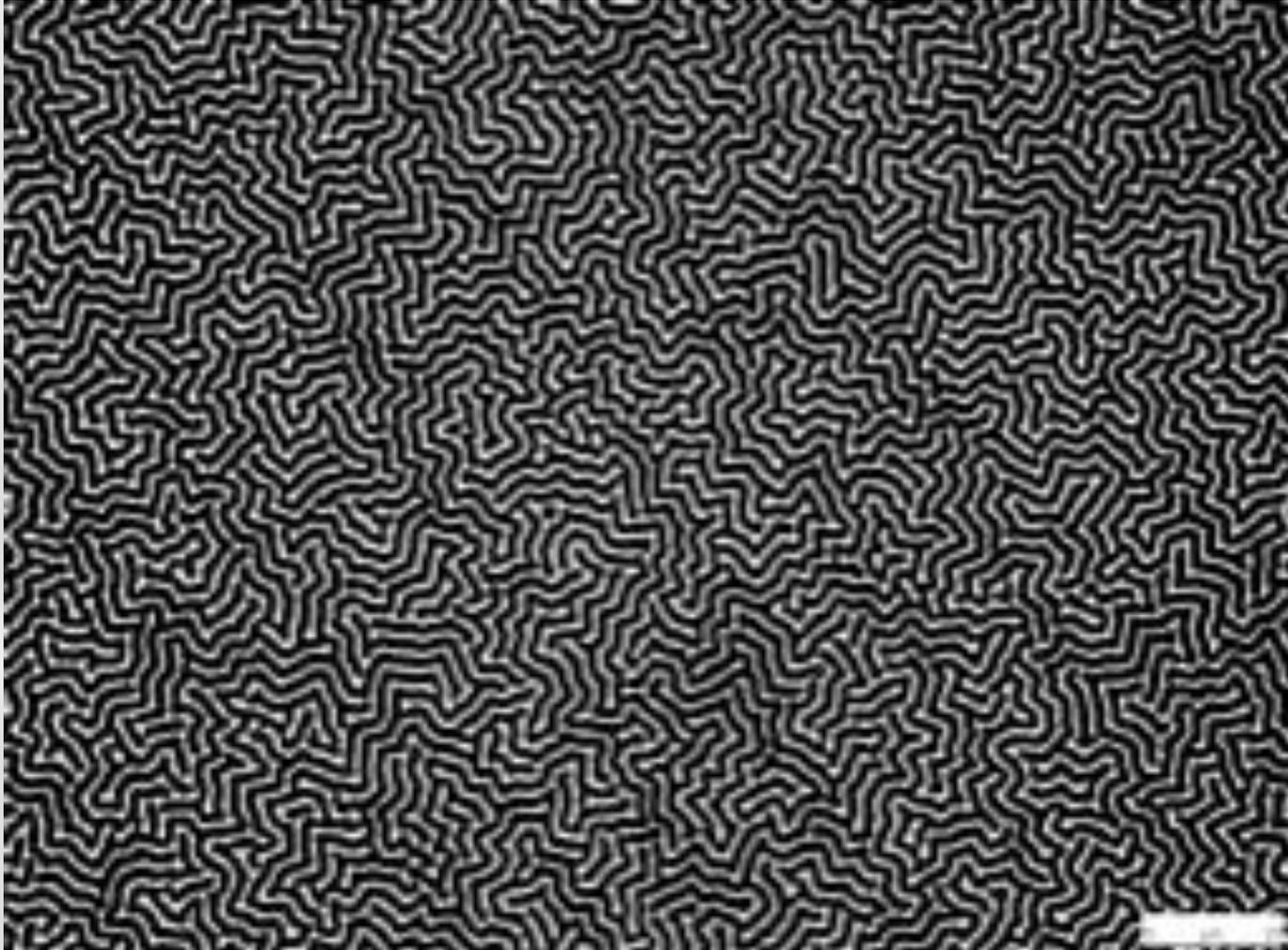
Chen, X. and J. Hutchinson, J. Appl. Mech.-T Asme, 2004



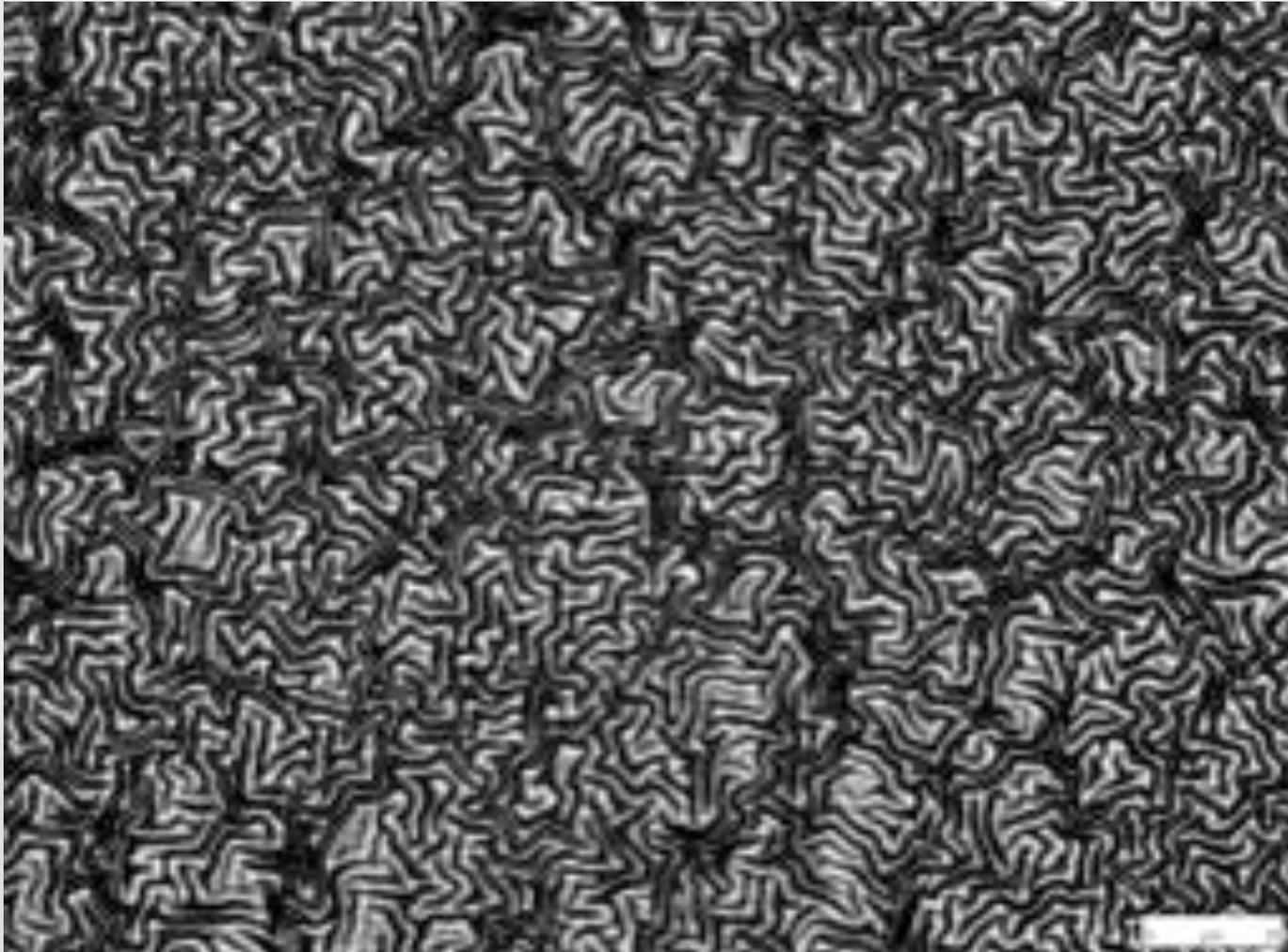
$$\frac{A}{\lambda_0} \approx \frac{\sqrt{1+\nu}}{\pi} \sqrt{|\varepsilon| - \varepsilon_c}$$



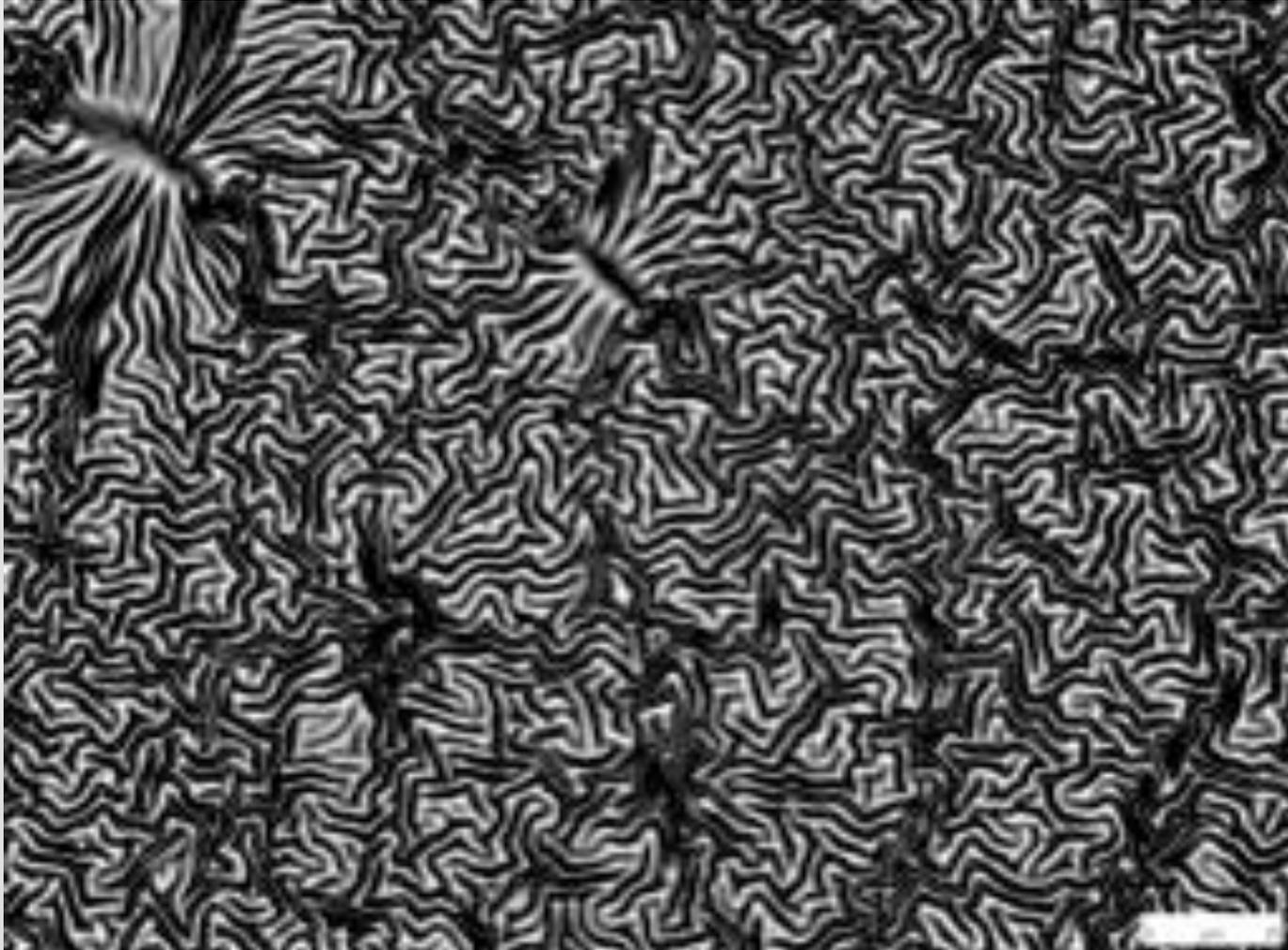
Fold nucleation



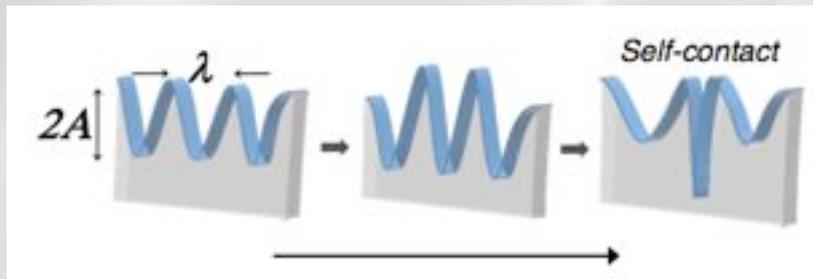
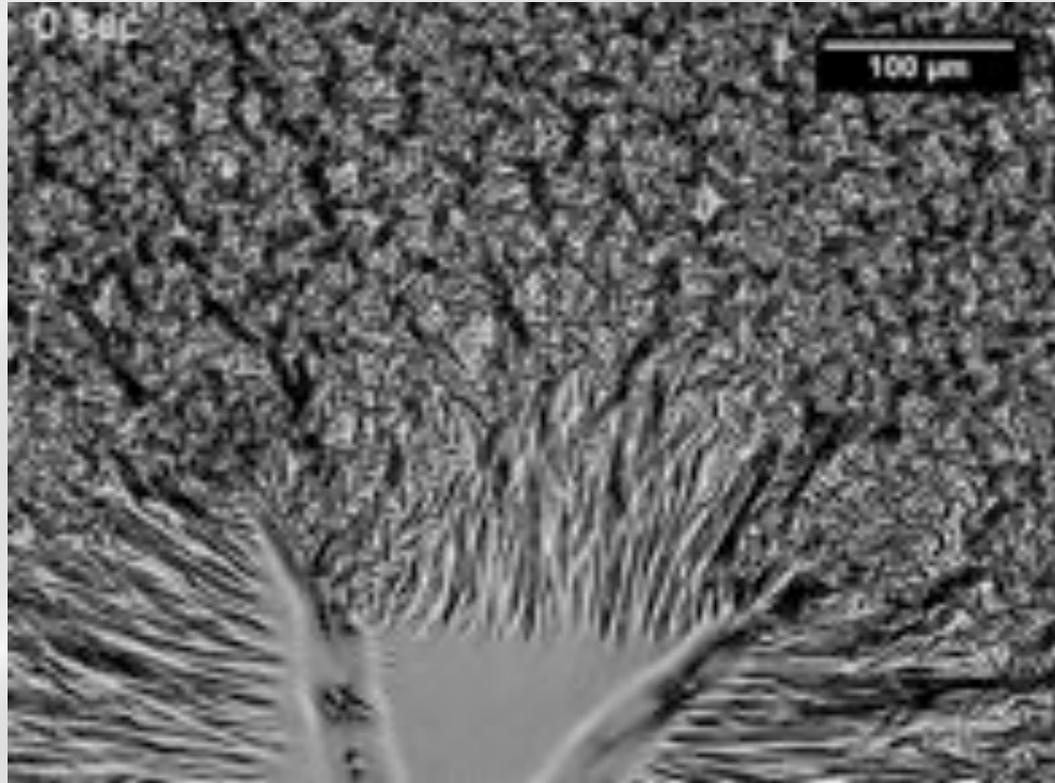
Fold nucleation



Fold nucleation



Nonlinear transition: Wrinkle-to-Fold



SEM of a fold cross section



Reticulated network formation

6 s

FOLD: NUCLEATION-GROWTH-NETWORK

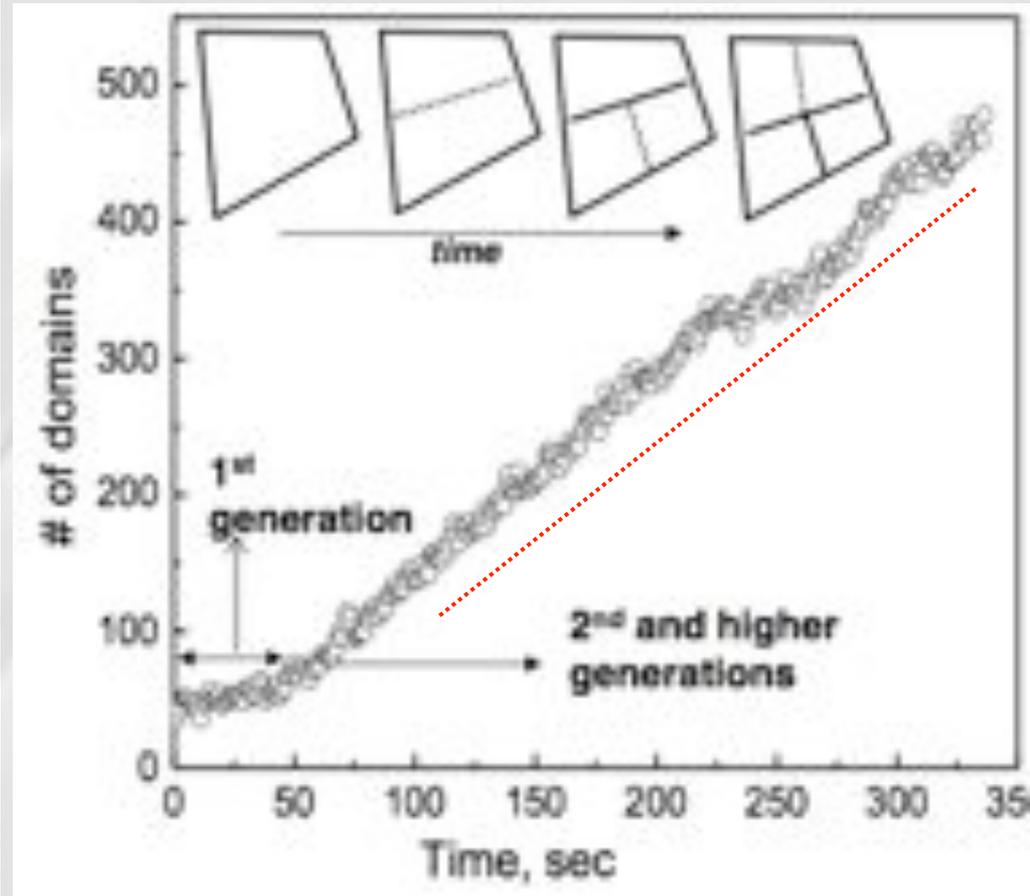
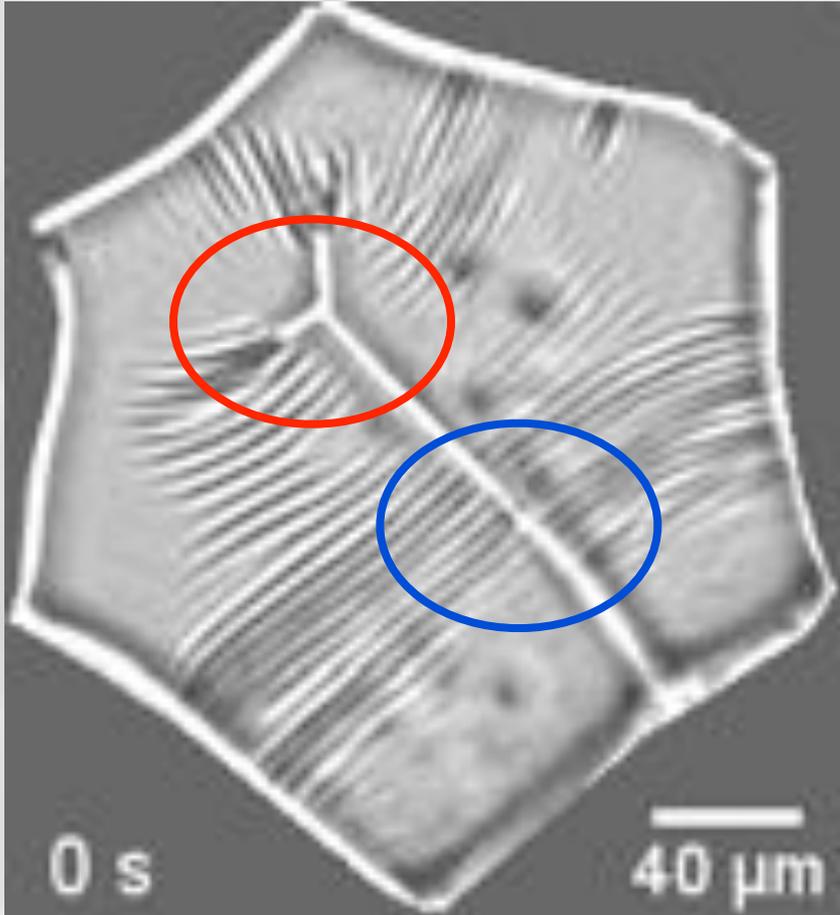


Pilnam, Abkarian* and Stone, Nature Materials 2011* 19

Hierarchical partitioning of the surface

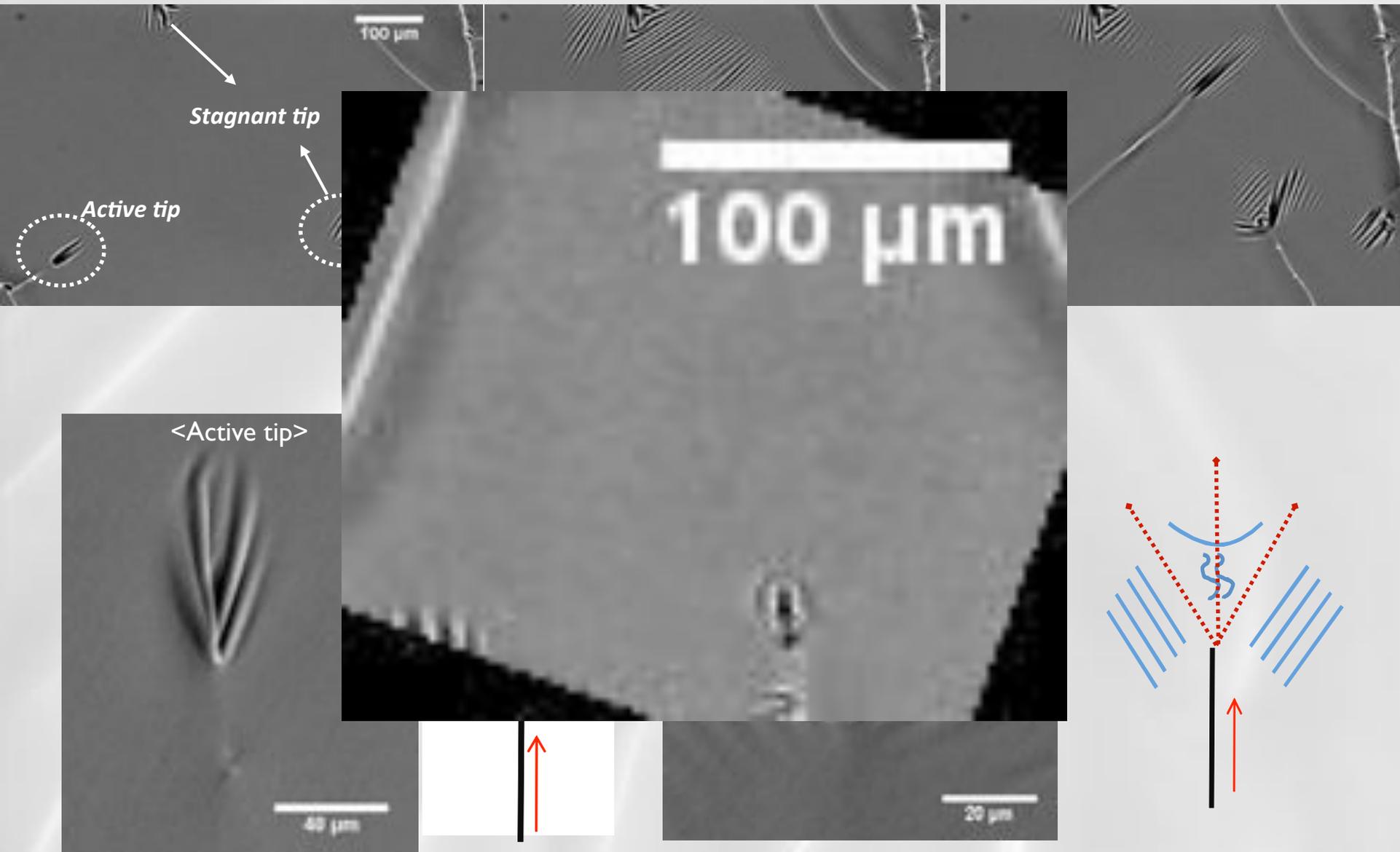
Terminal and segmental branches

Linear growth rate
due to the self-regulation



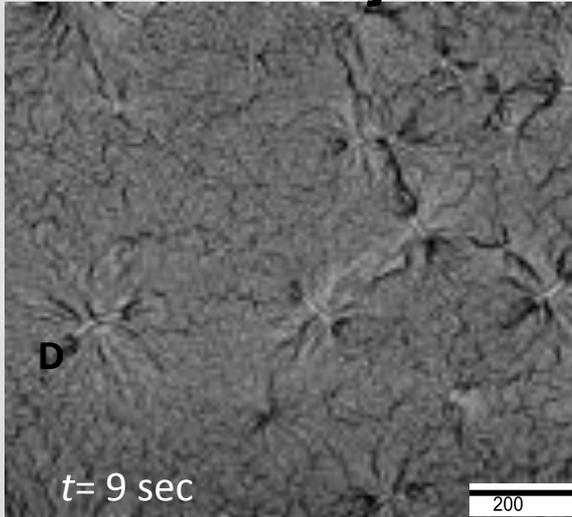
Self-regulation process

$\Delta t = 5 \text{ sec}$

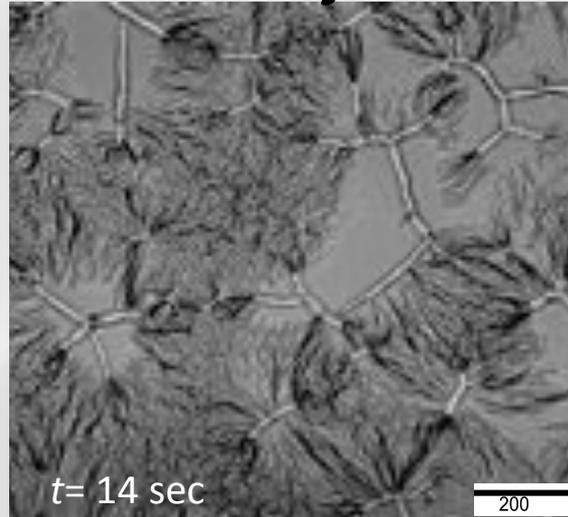


Communication and networking

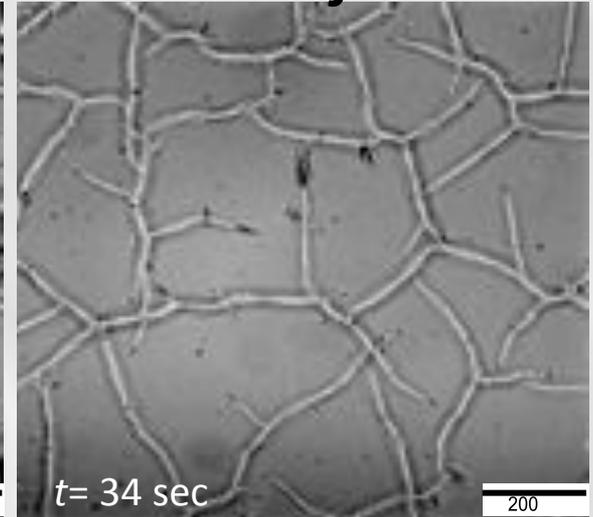
Nucleation of Folds



Growth of Folds

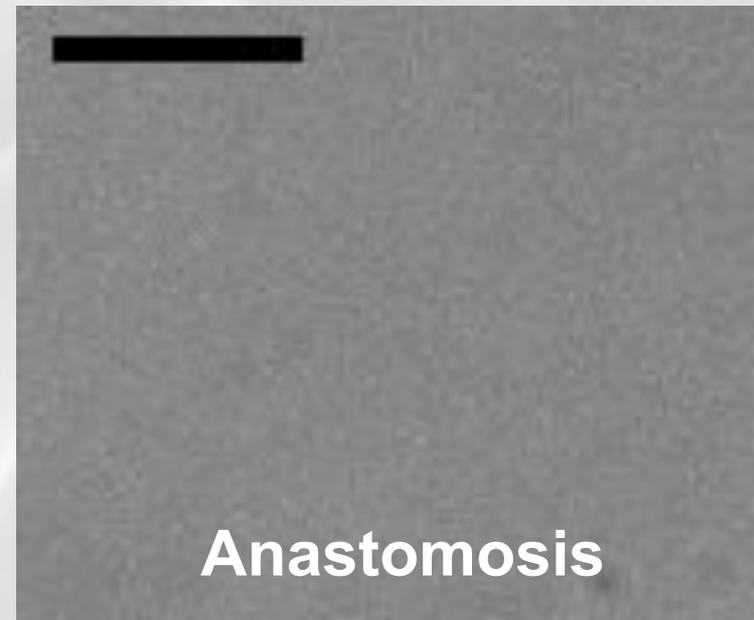
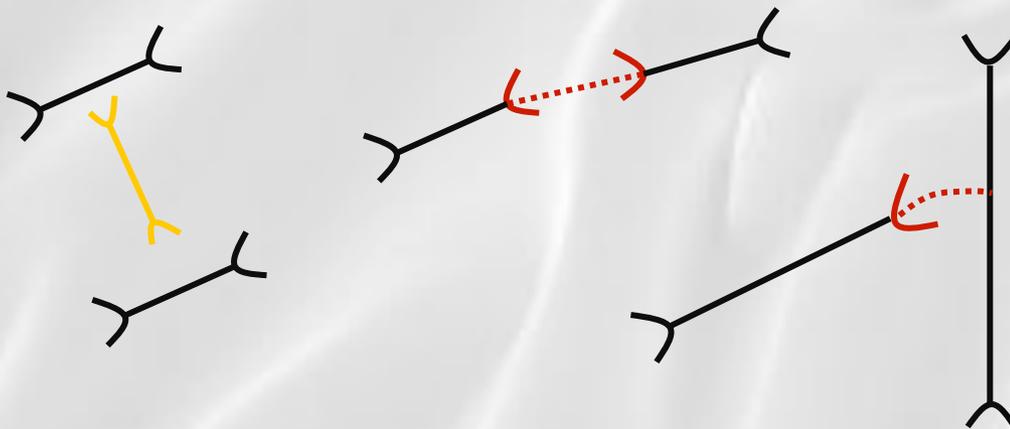


Formation of Network



Time →

➤ *How does it form closed domains?*



Self-regulation process: fold-fold interaction

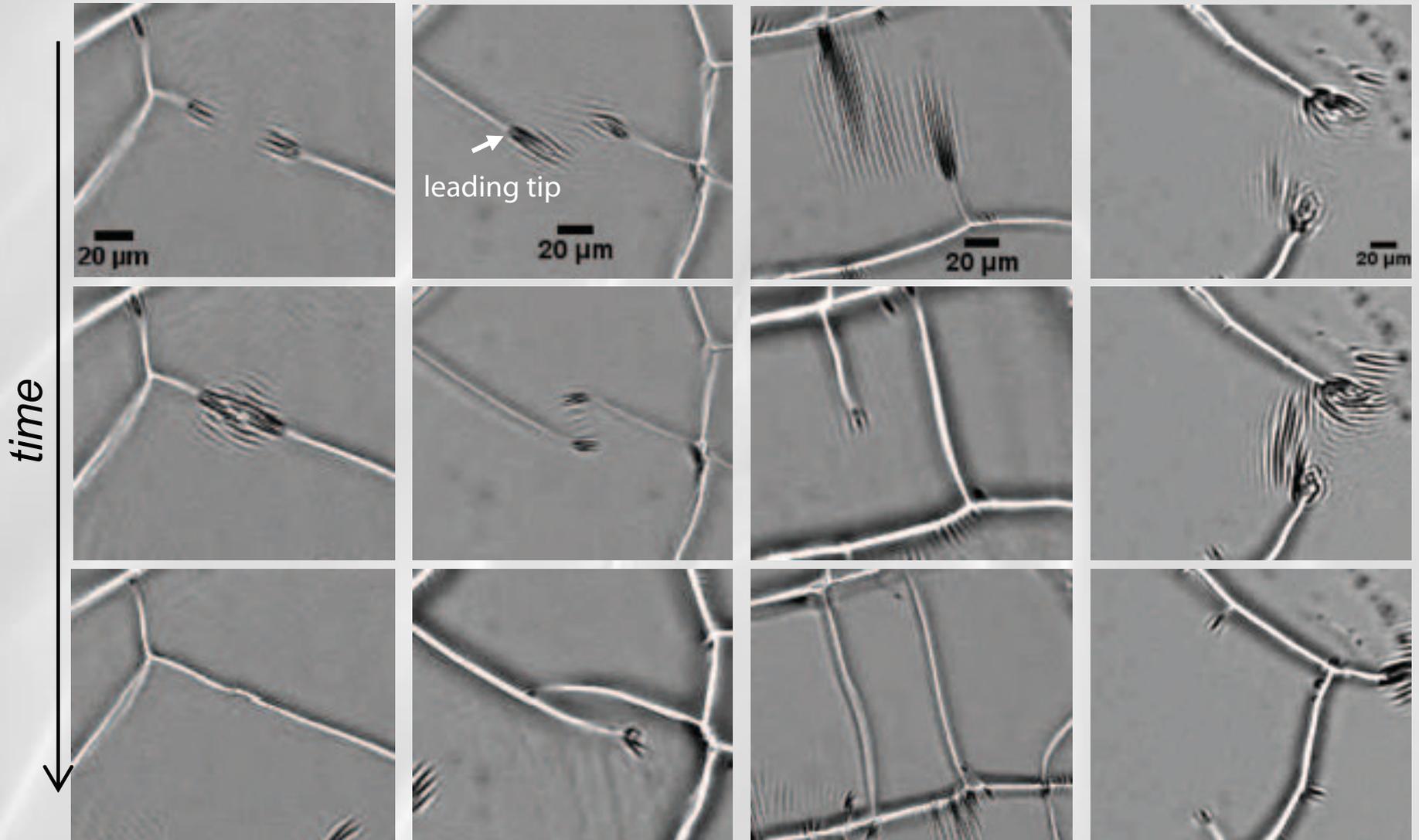
Bending

a) Head-on coalesce

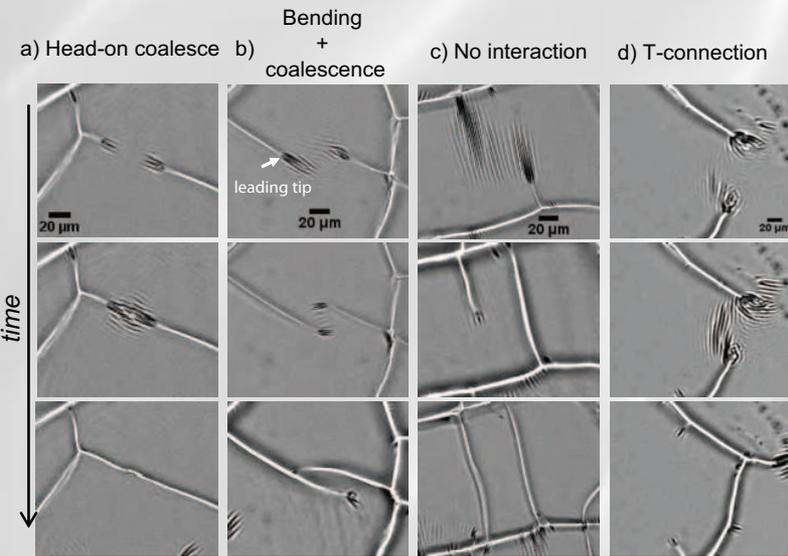
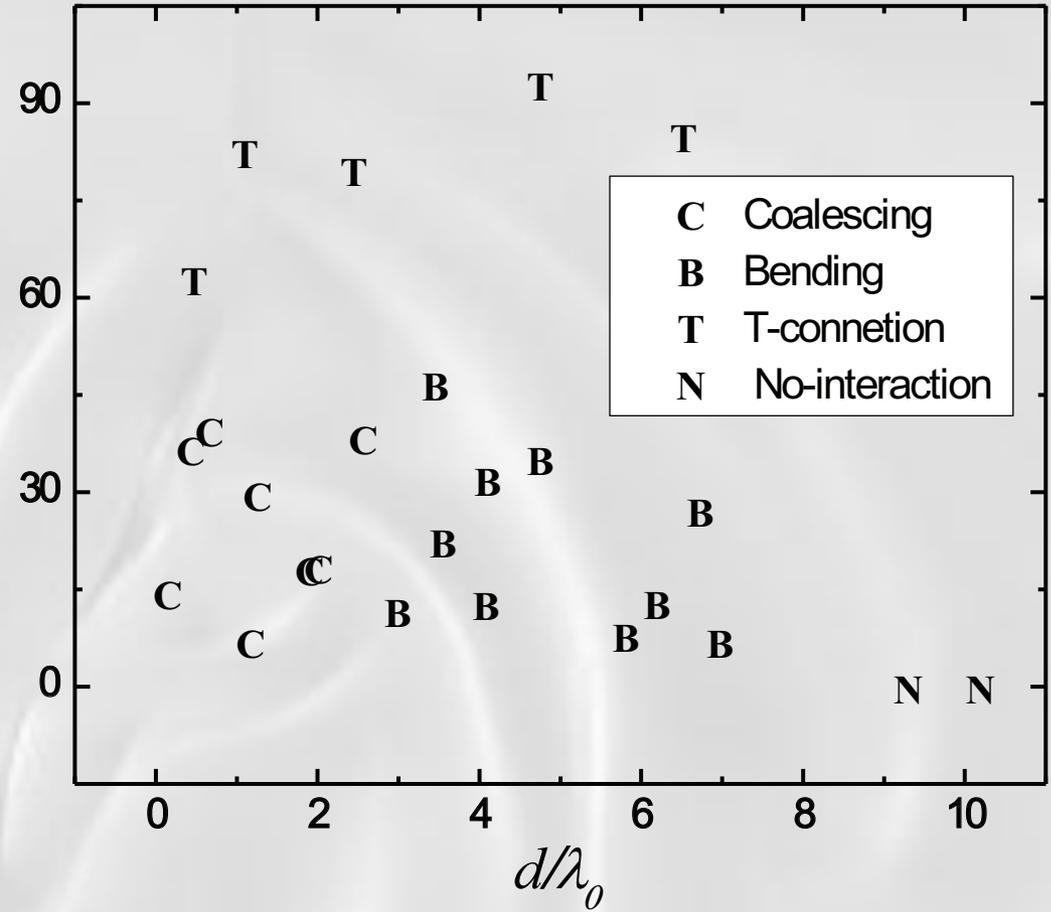
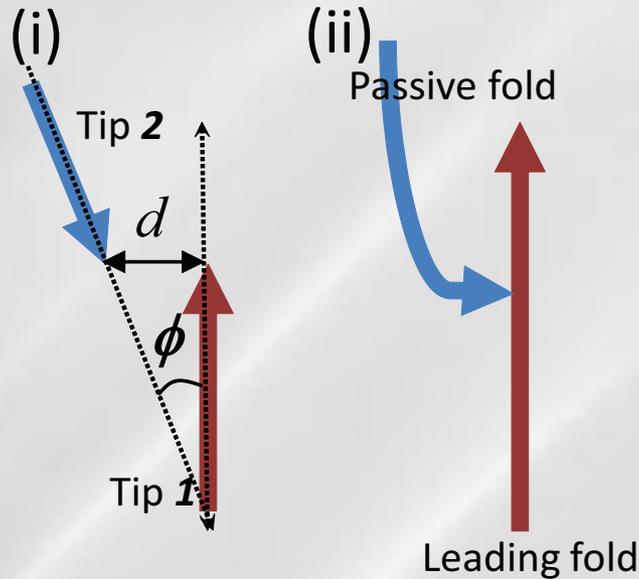
b) ⁺
coalescence

c) No interaction

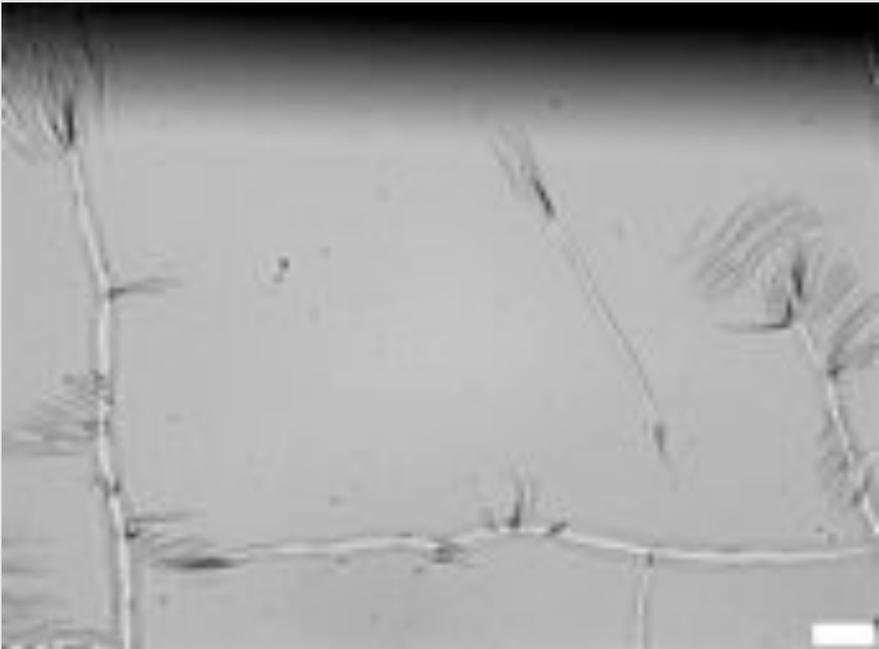
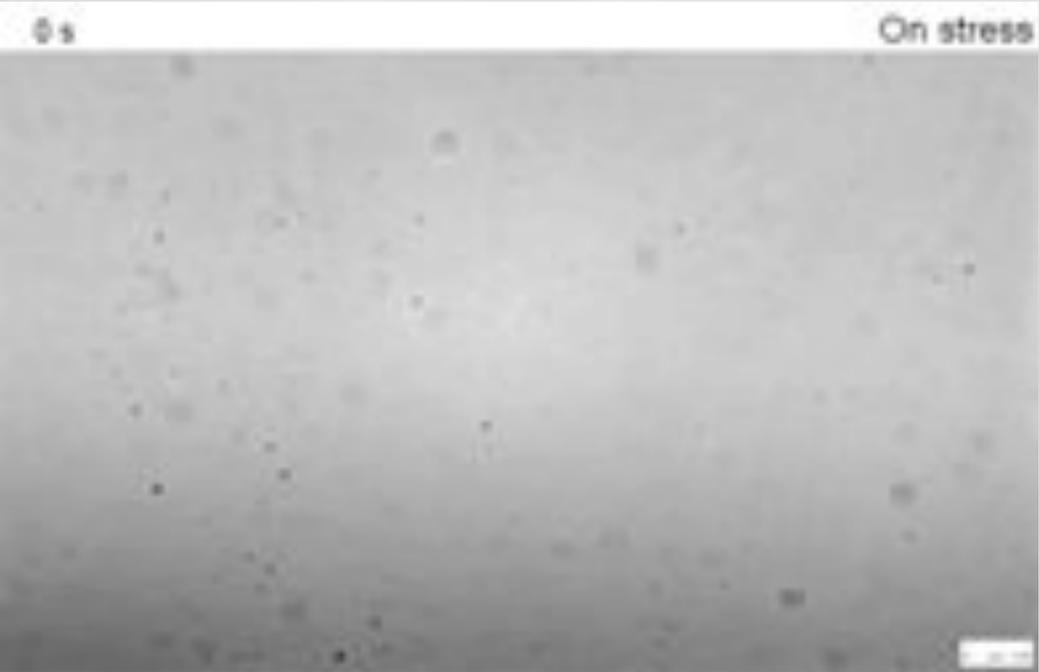
d) T-connection



Self-regulation process: fold-fold interaction



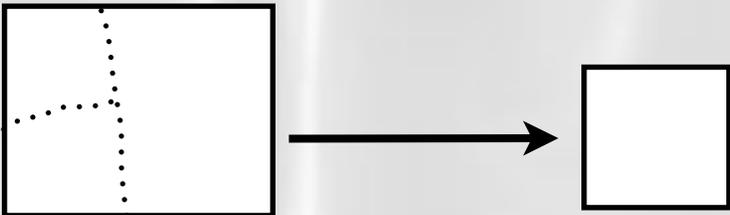
Boundary Effect and defects density



Isotrope



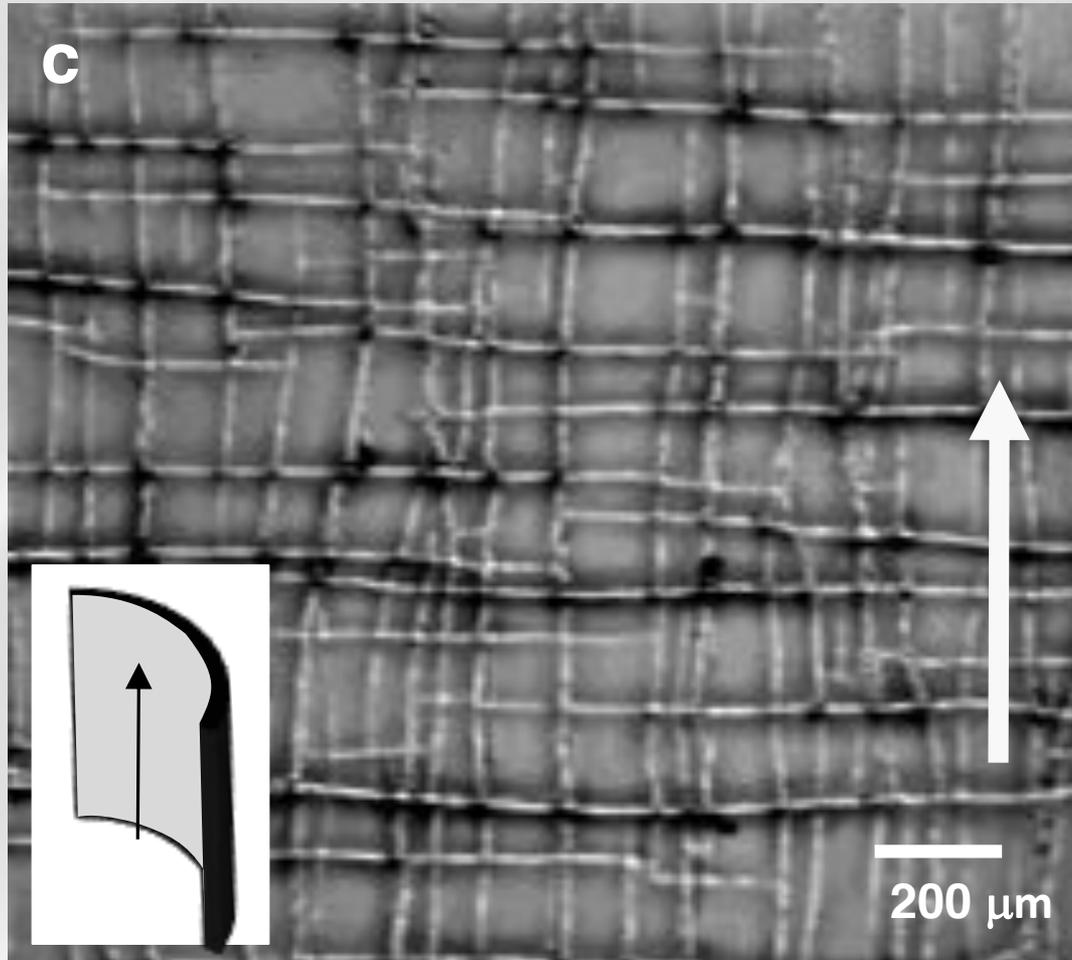
With parallel folds



analogous to cracks

Bohn et al., 2005

Other features: playing with geometry



Conclusion

Formation of a **network** under a 2D compressive stress in multilayers

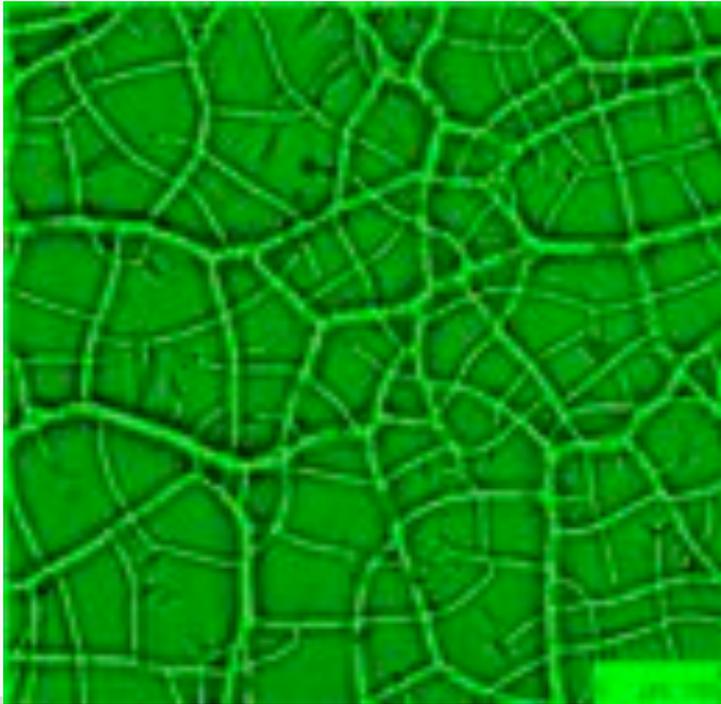
Result of **repetitive wrinkle-to-fold transitions**:

Folds propagate, frequently branch and intersect with each other in a **self-regulated manner**.

The **hierarchical division of space** is fundamentally linked to the **tensorial nature of the stress field** and the **local and anisotropic release of stress via fold localization**

Can we say something about venation ?

Folds



Schizophragma hydrangeoides

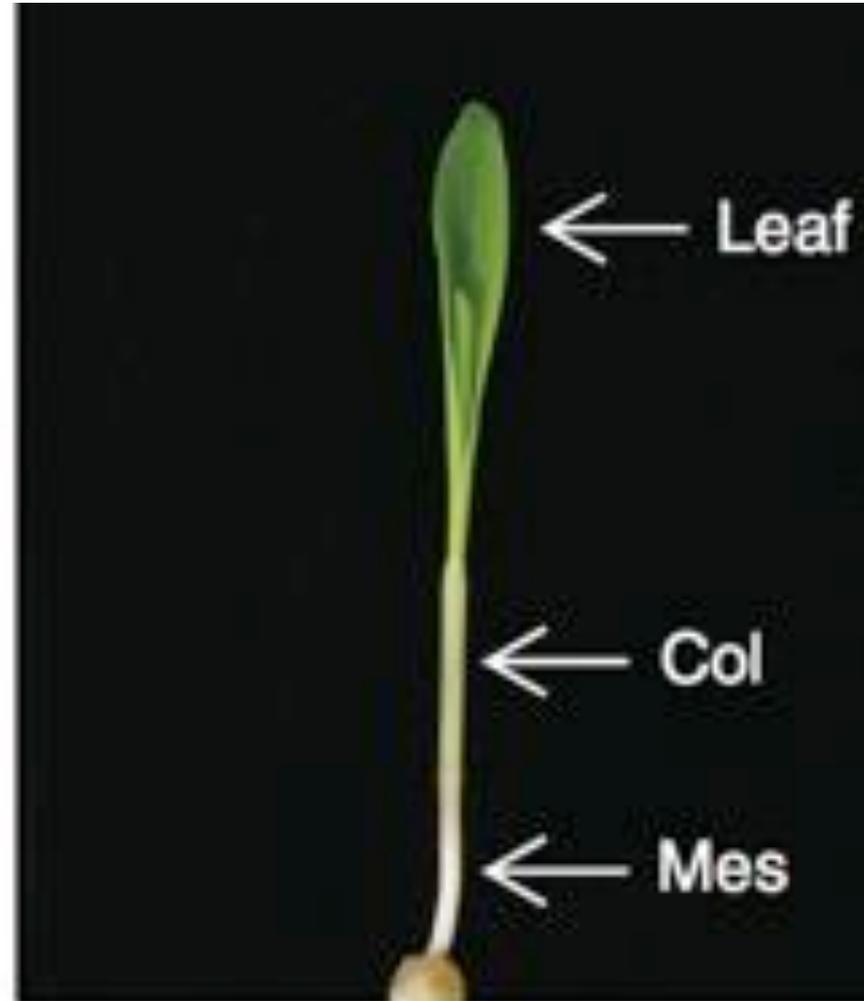
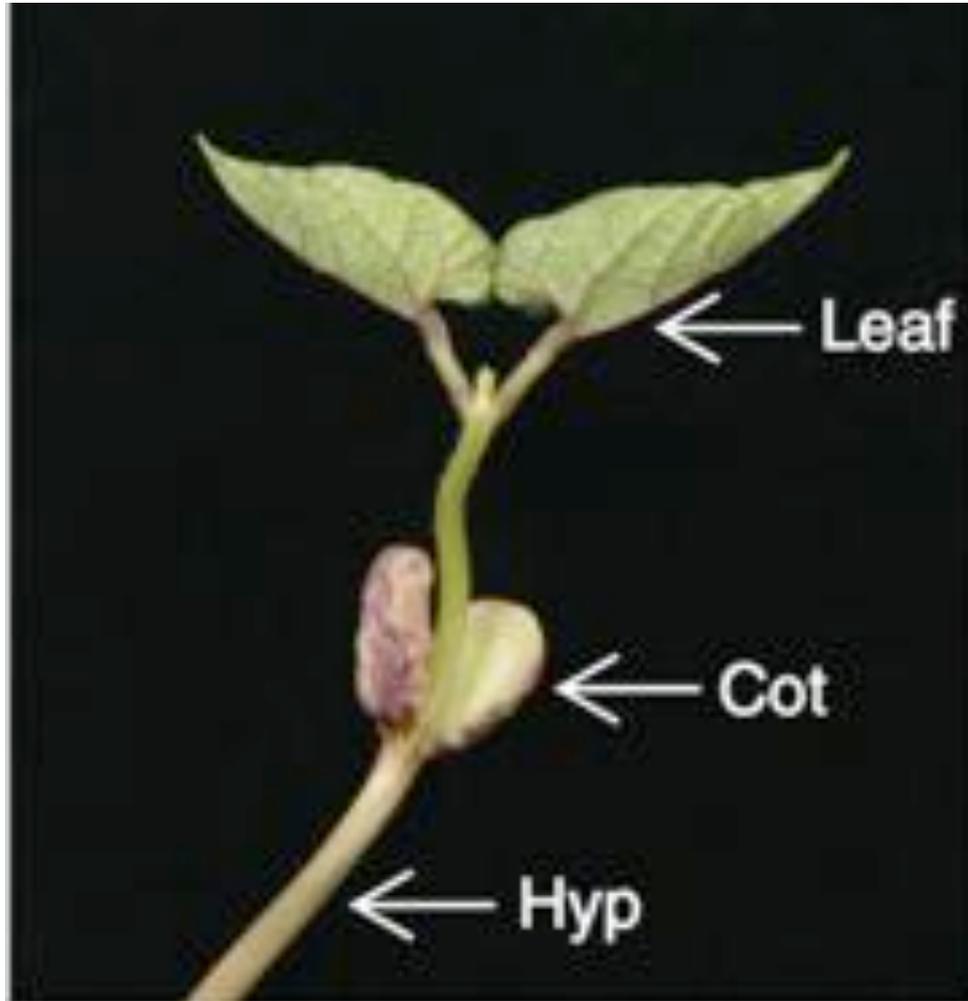
reticulated

Leaves venation architecture

Two groups of flowering plants

Dicotyledons

Monocotyledons



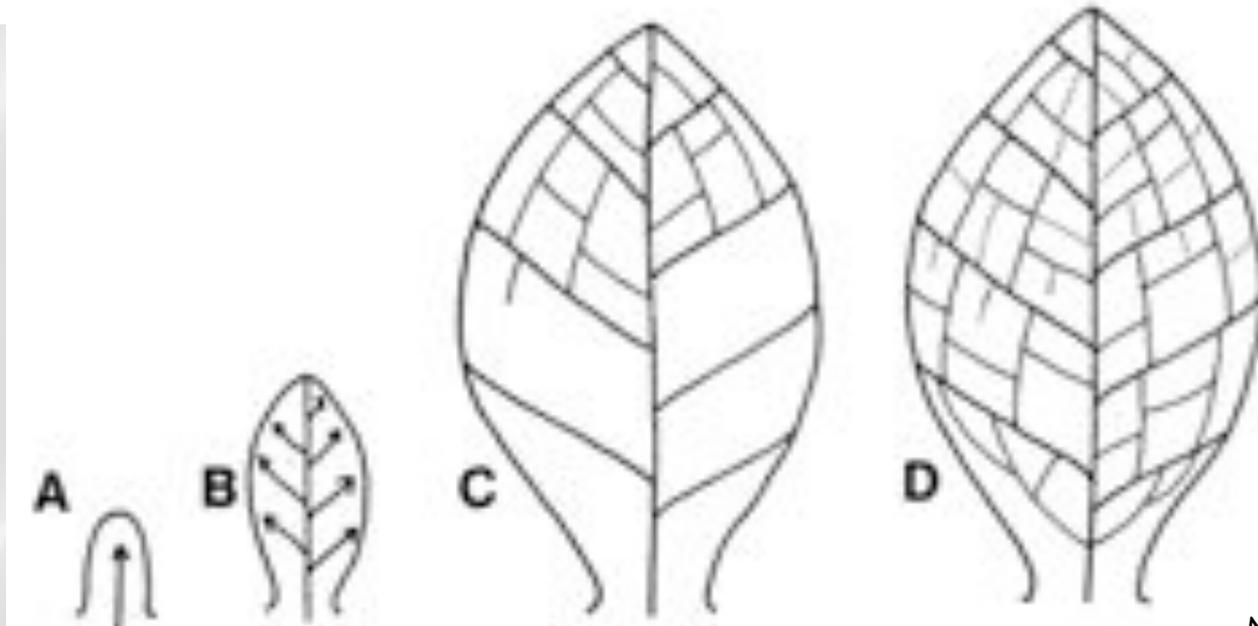
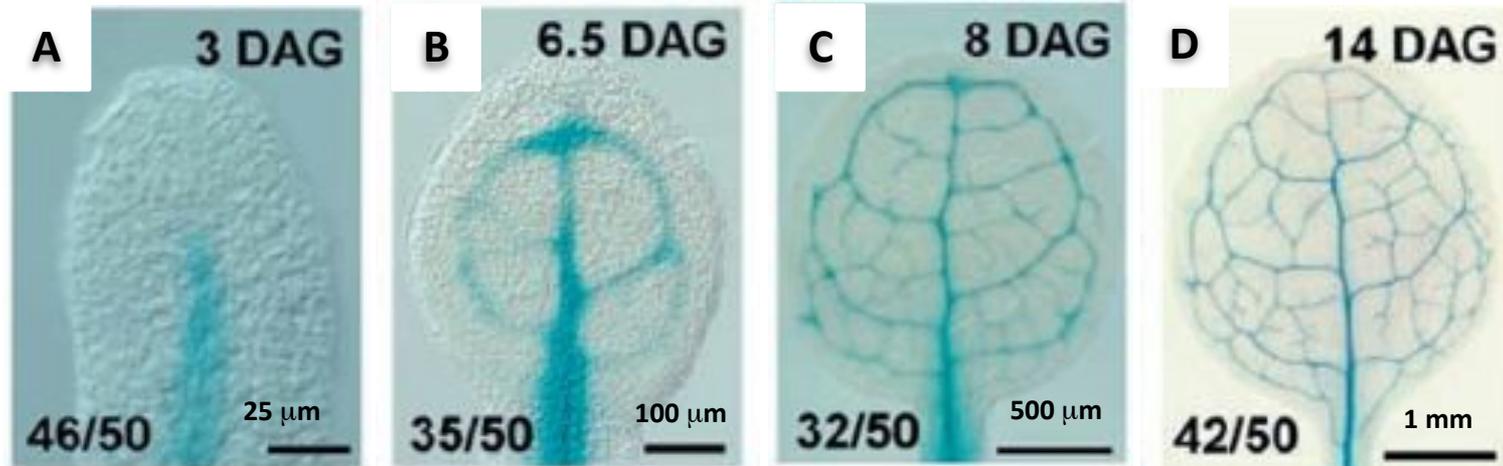
McSteen, 2010

Venation formation and architecture

Hierarchical division of space

Network of veins

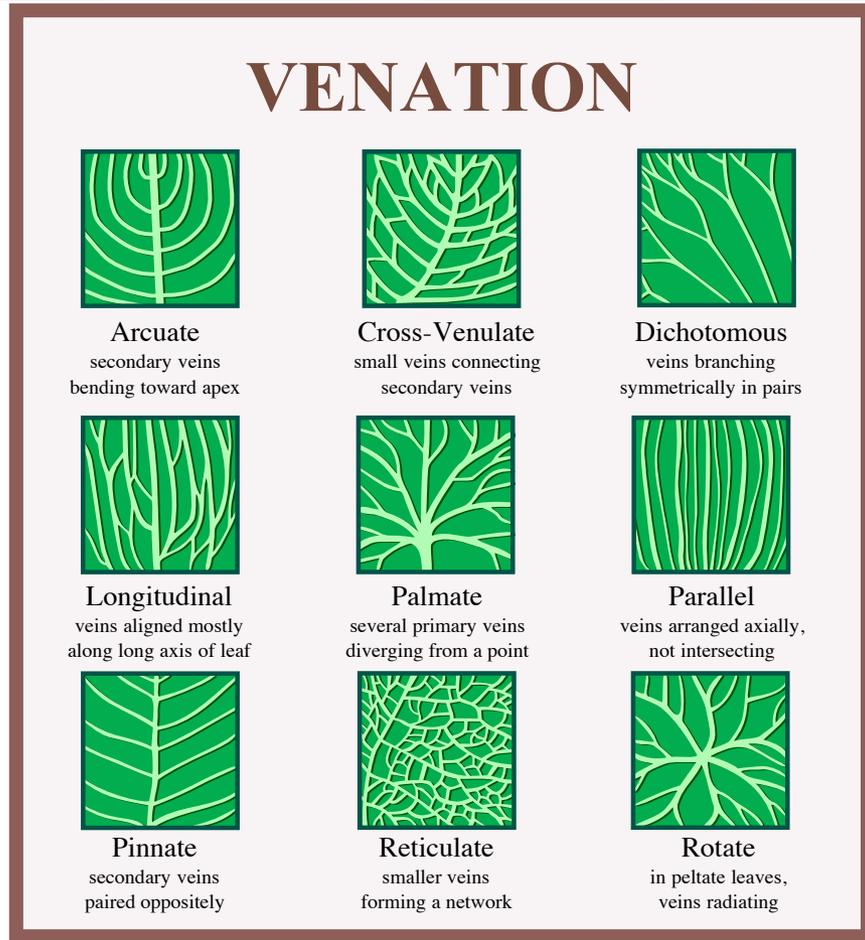
Scarpella et al, *Development* (2004)



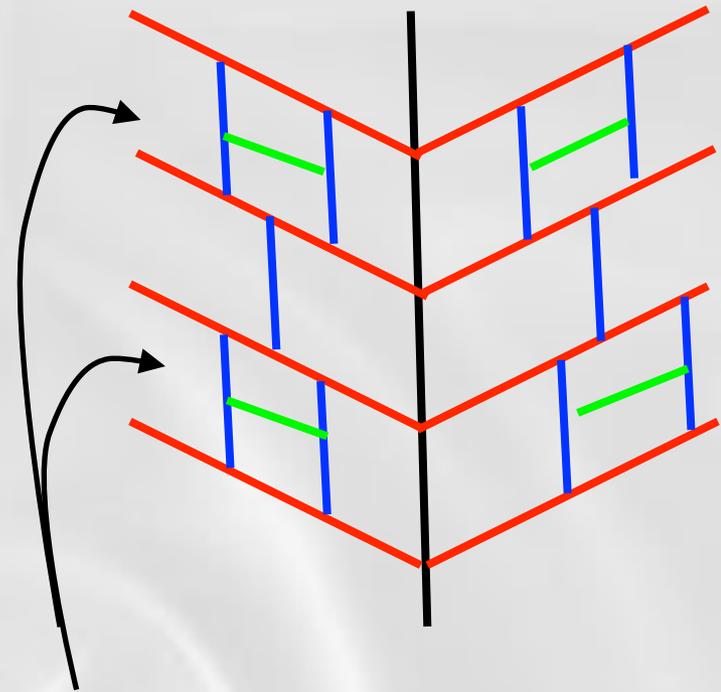
Nelson and Dengler 1997

Leaves venation architecture

1st and 2nd generation conserved
in the plant



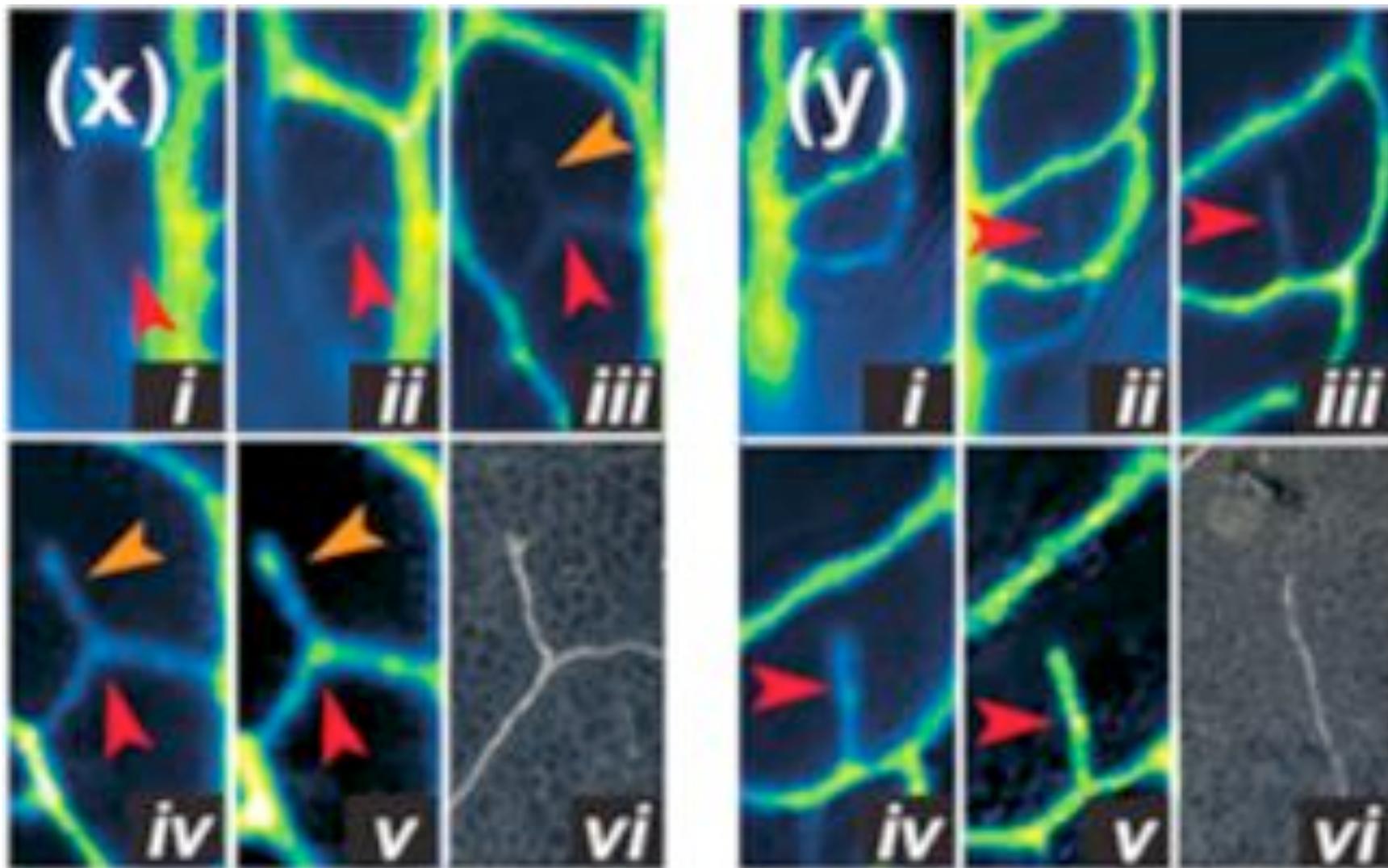
wikipedia



3rd, 4th...generations
variable from leaf to leaf

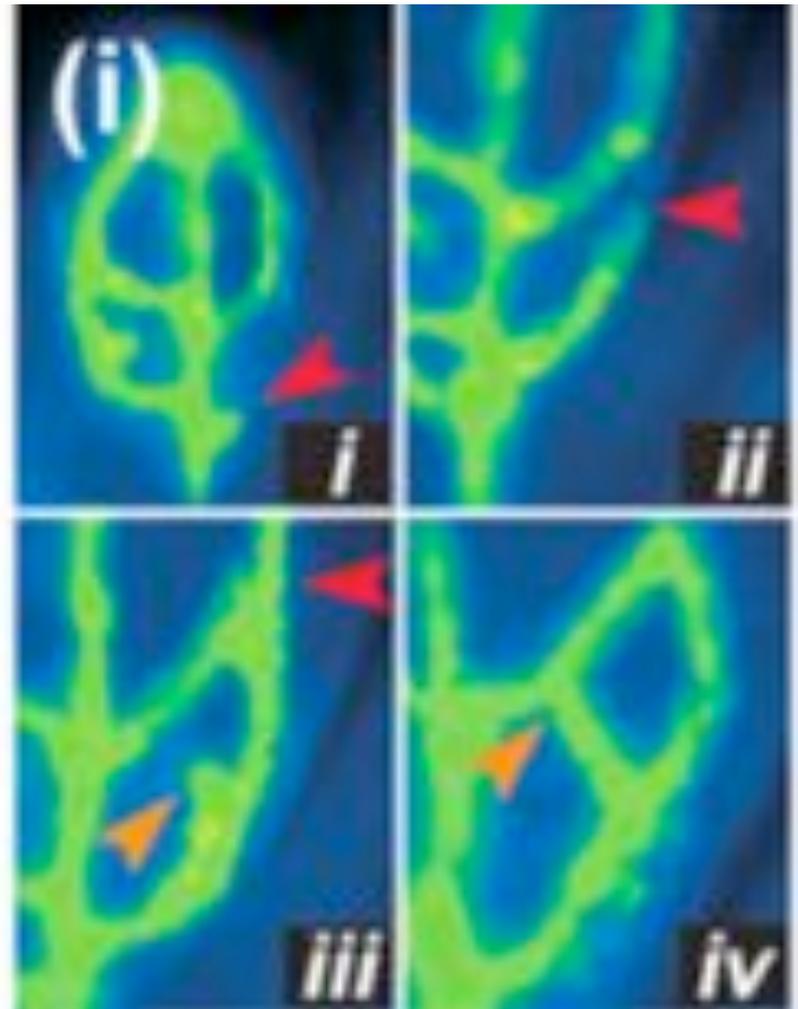
Self-organized process
not only genetics ?

Branching



Sawchuk et al. 2007

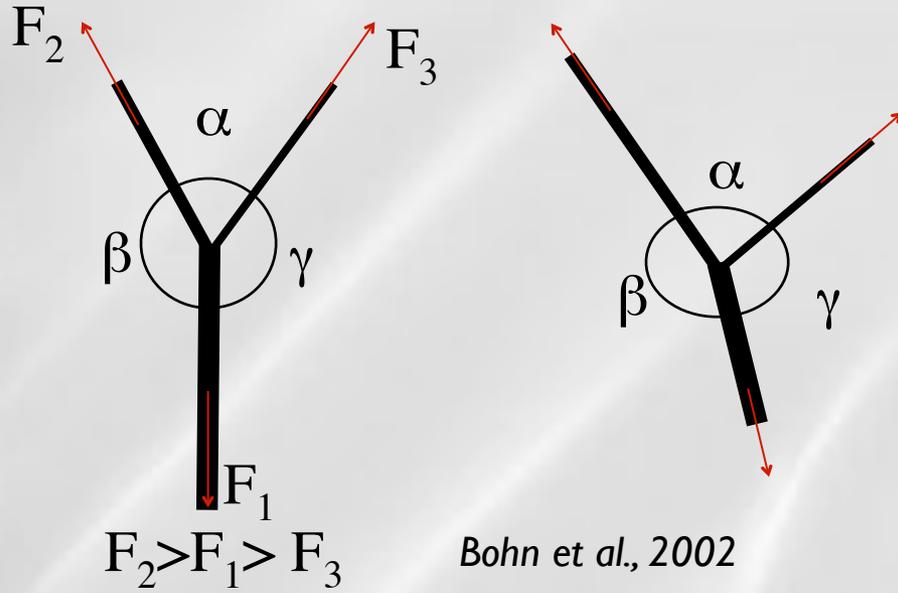
Reorganization



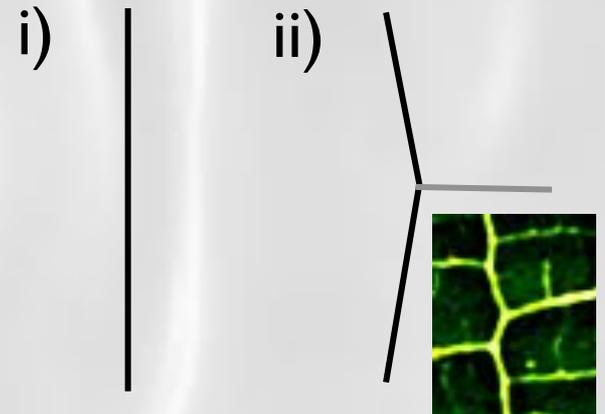
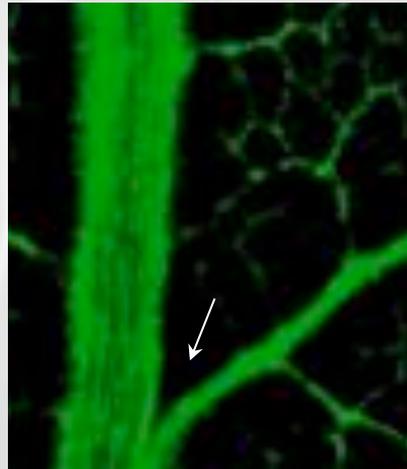
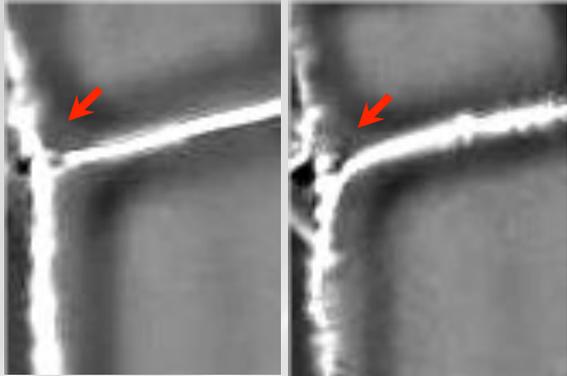
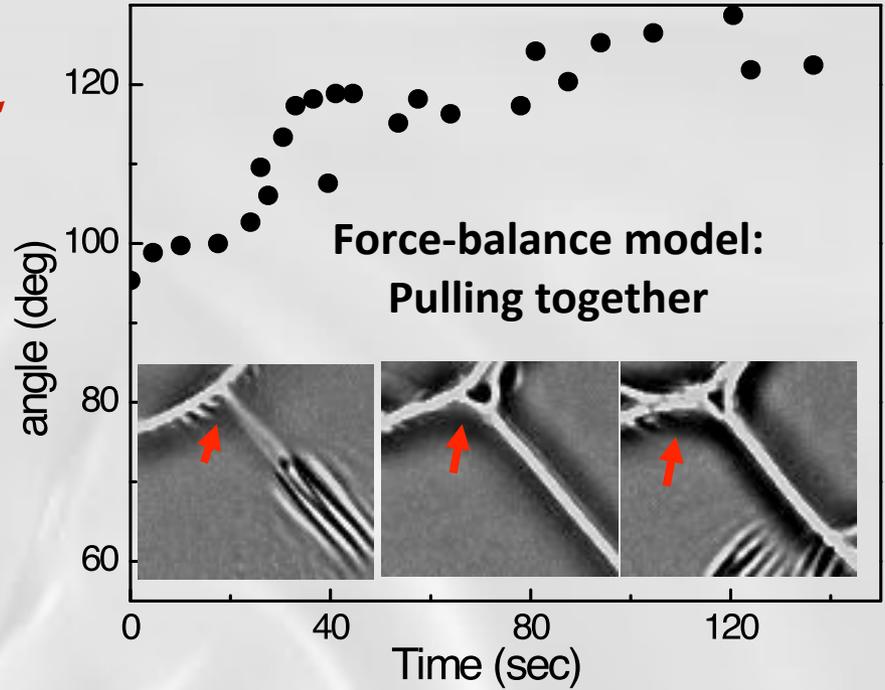
Sawchuk et al. 2007

Not only static but also dynamic

Time-dependent deformation: Remodeling

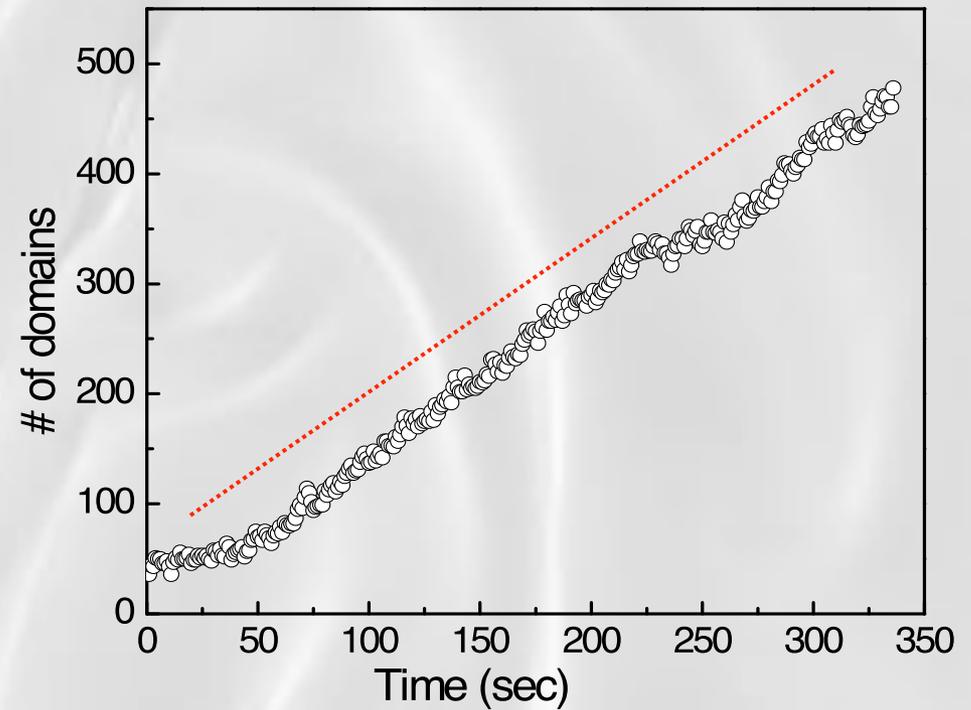
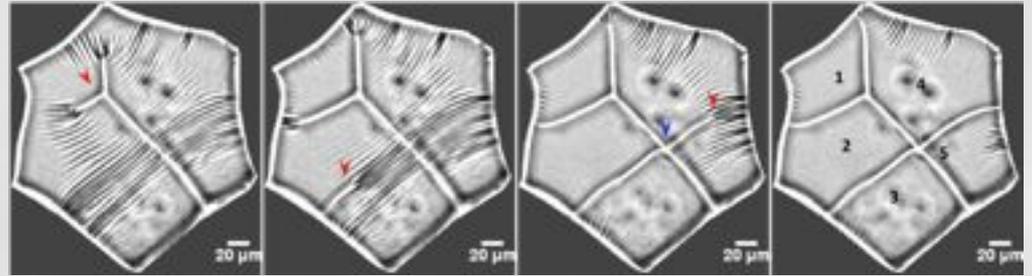
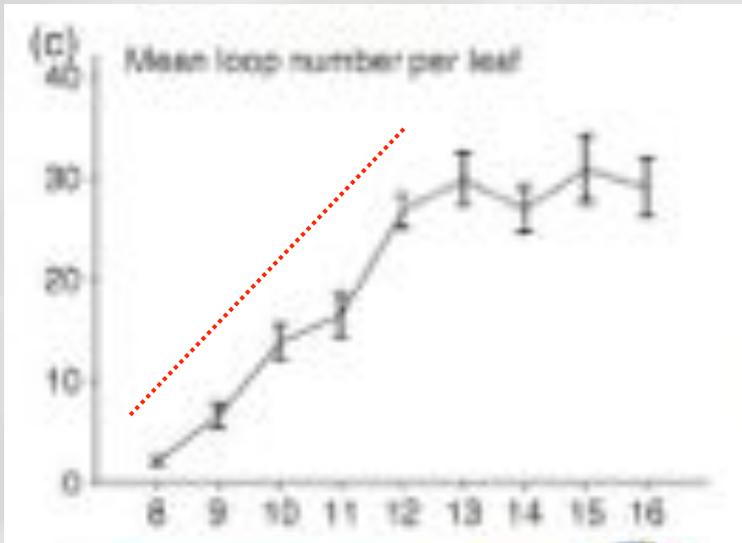


Bohn et al., 2002



Not only static but also dynamic

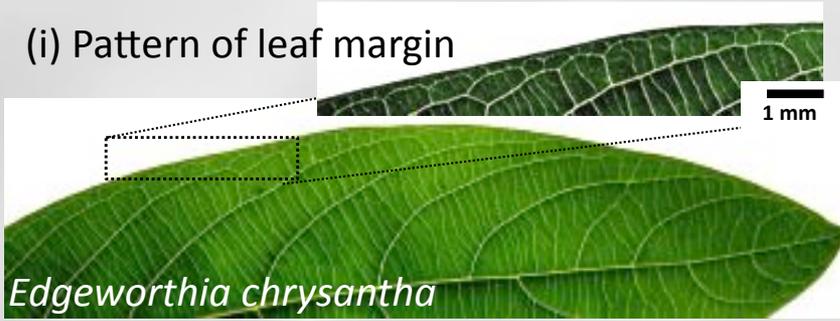
Linear growth rate due to the self-regulation



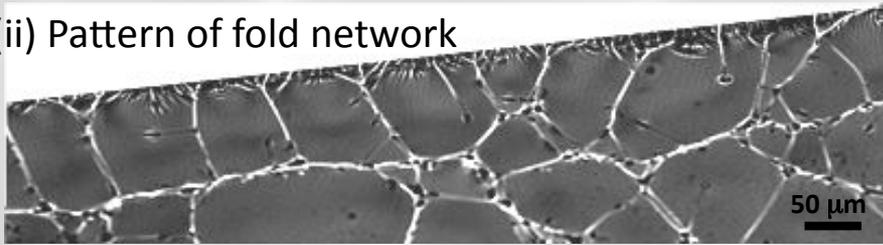
A. Rolland-Lagan, *The Plant Journal* (2009)

More analogies with venation of leaves

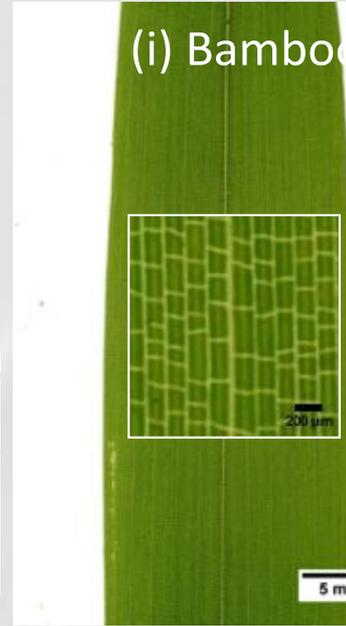
(i) Pattern of leaf margin



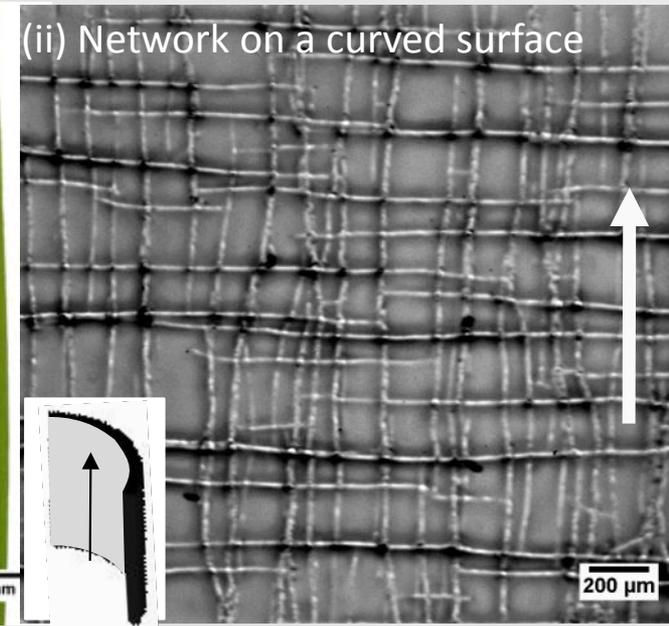
(ii) Pattern of fold network



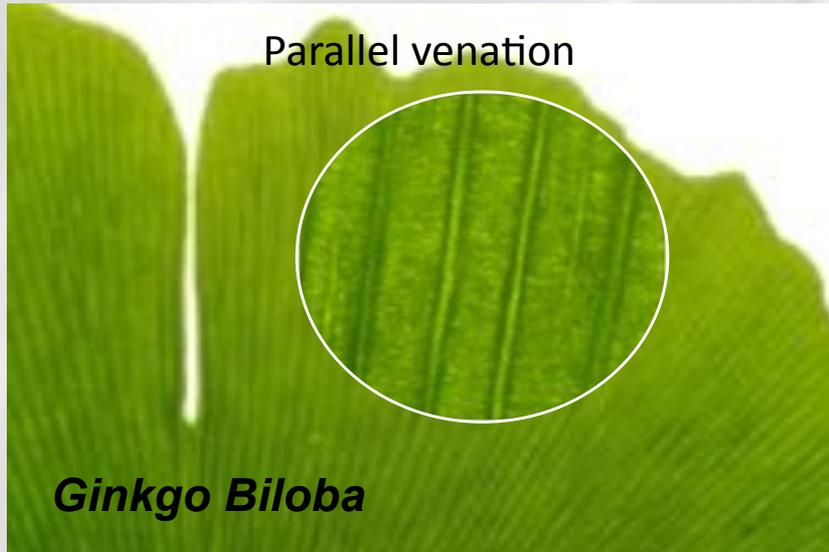
(i) Bamboo



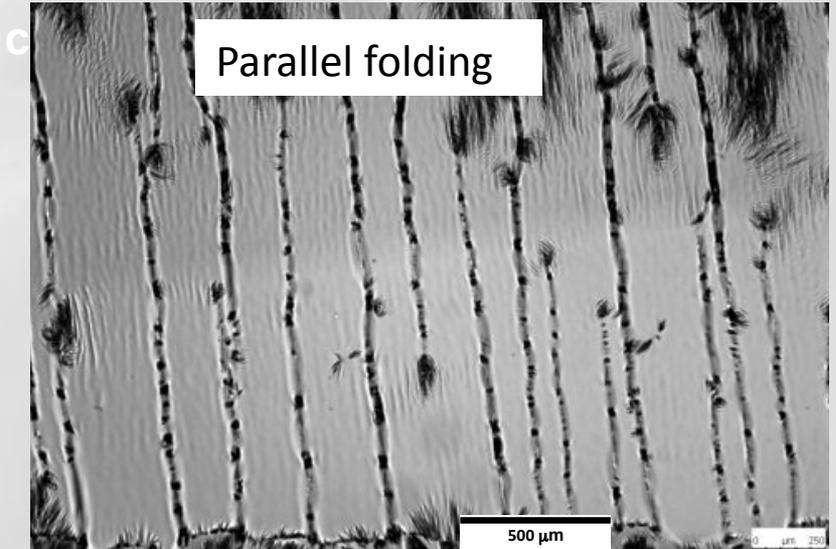
(ii) Network on a curved surface



Parallel venation

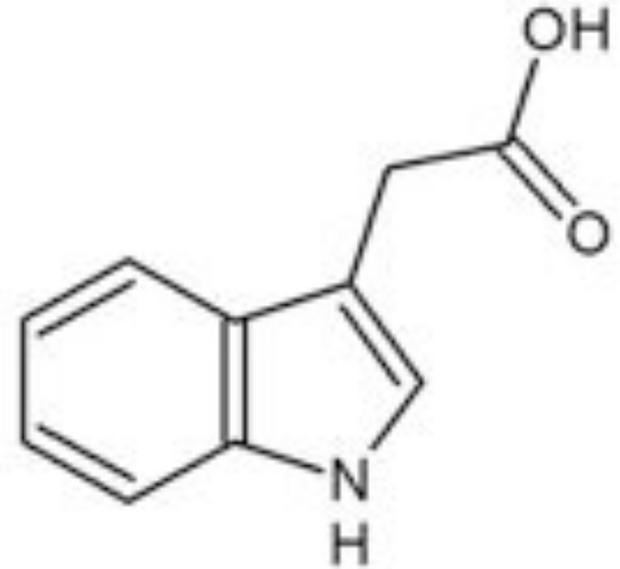
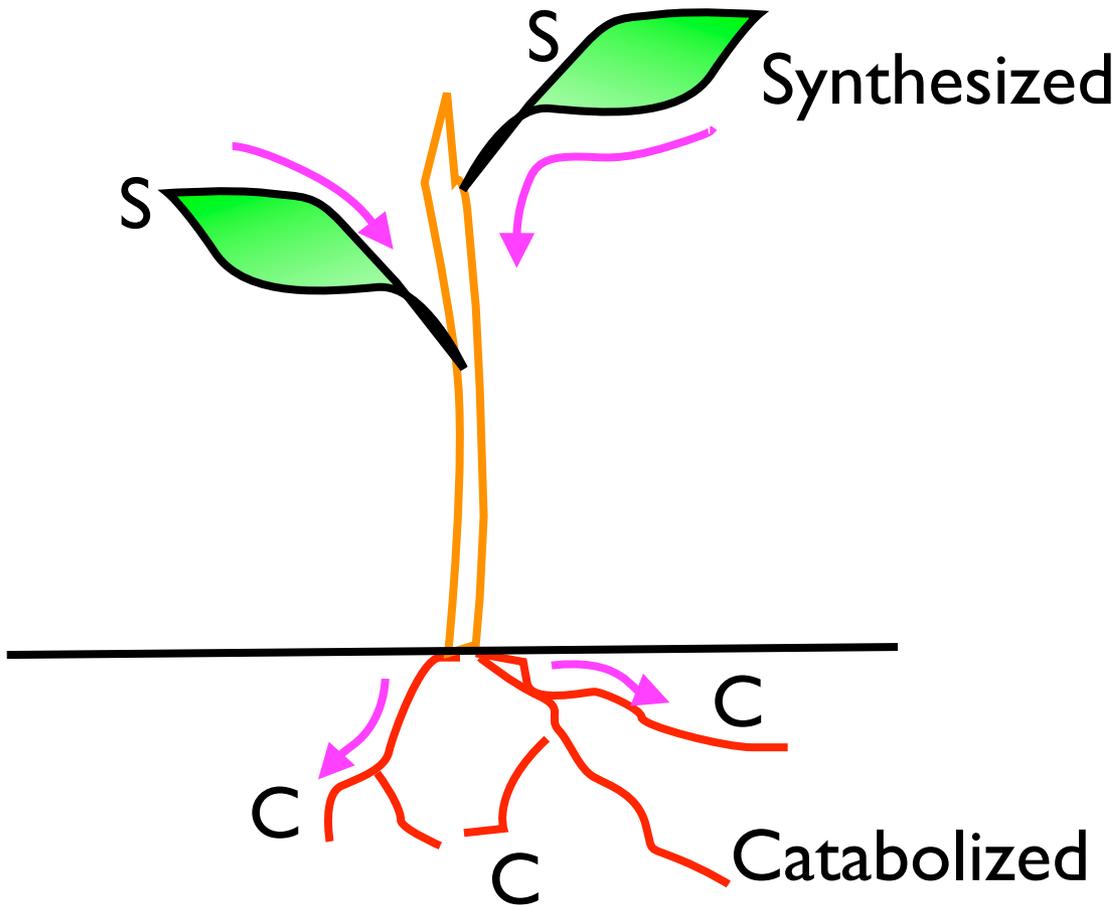


Parallel folding



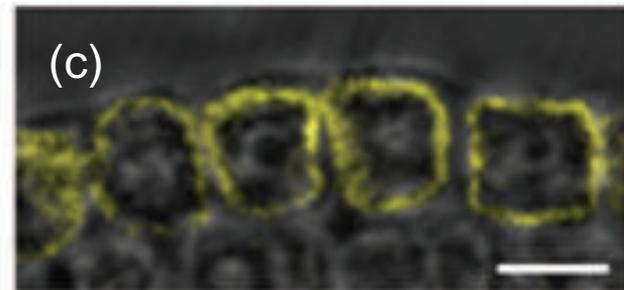
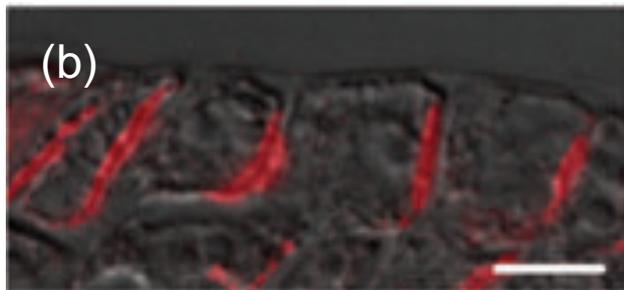
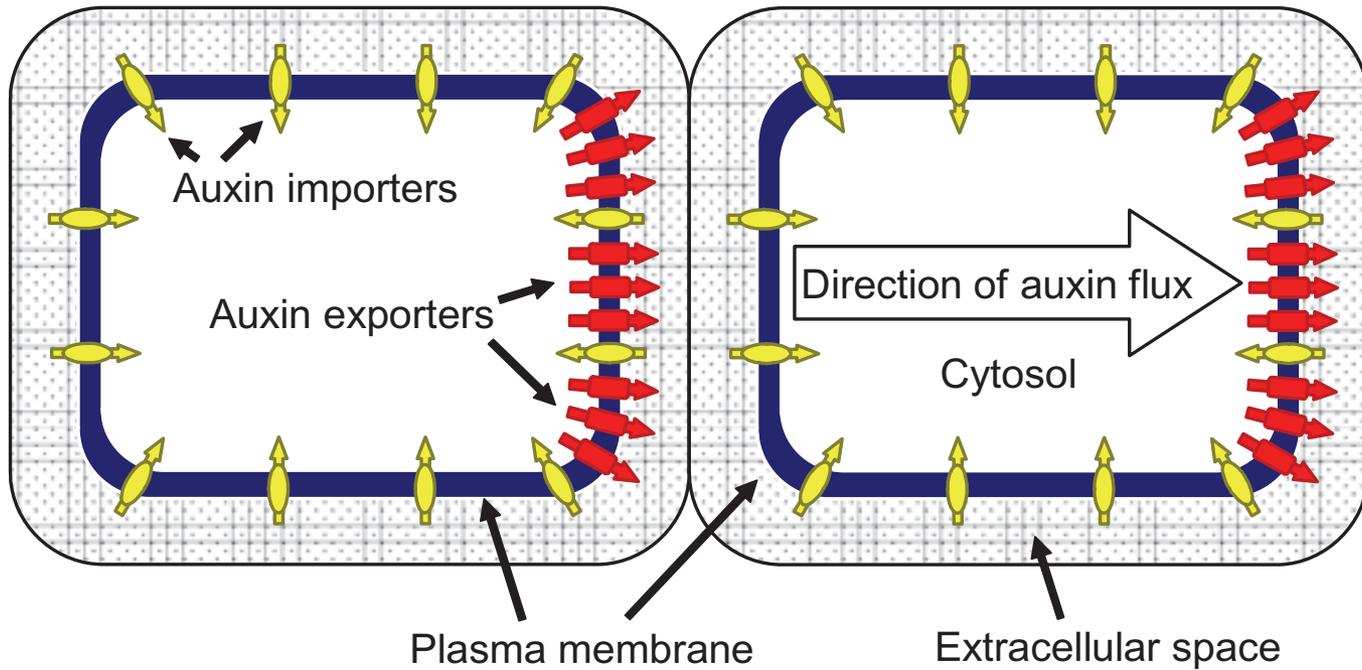
Growth hormone Auxin

from the greek *auxien* - «to grow»



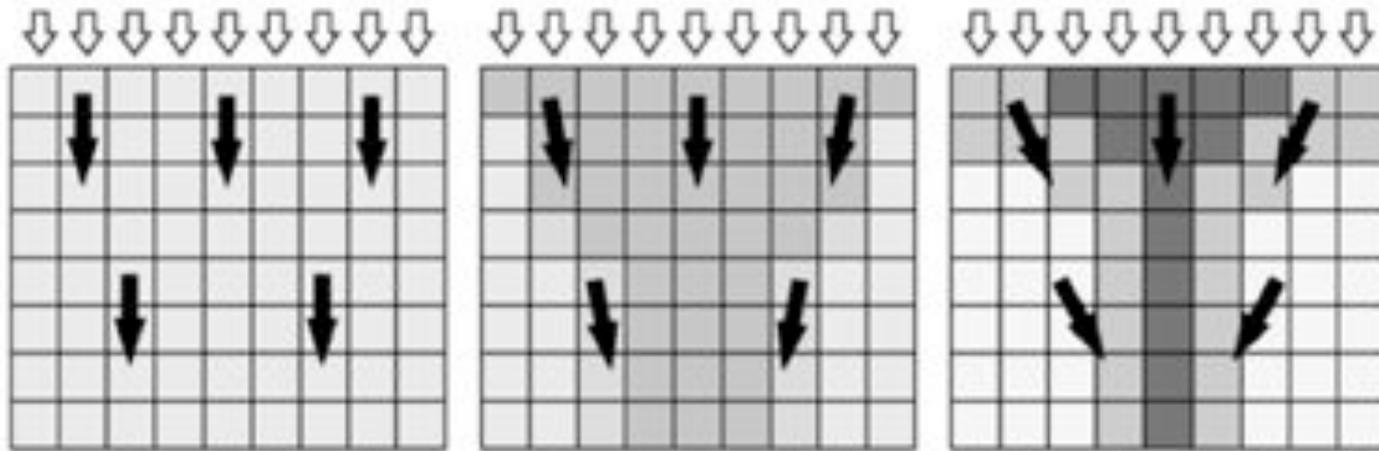
[indole-3-acetic acid](#) (IAA)

Growth hormone Auxin

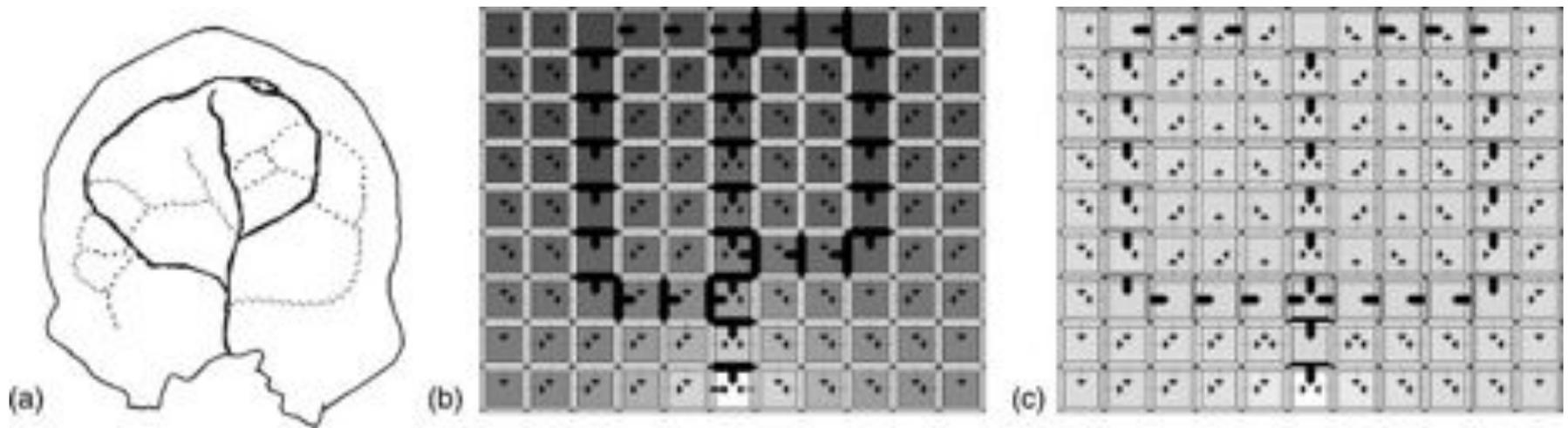


Smith and Bayer, *Plant, Cell and Environment* (2009)

Sachs Canalization hypothesis



Sachs, 1981



Rolland-Lagan

Symphony of sources and sinks

To model branching and loops: synchronization of sources and sinks

Range of hypothesis on Auxin production and detection of cells
with no clear experimental data

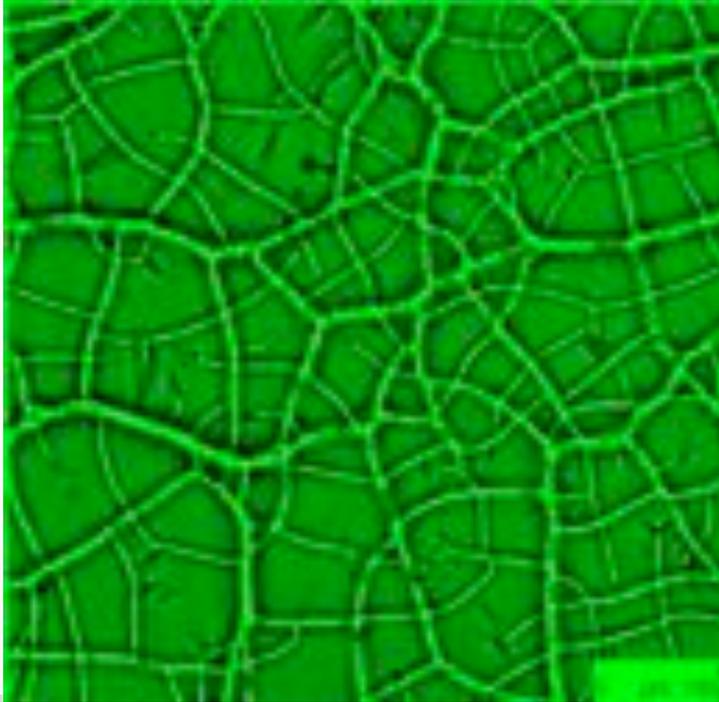
Where and when Auxin is produced by cells ?

How cells perceive Auxin differences and/or fluxes ?

Santos et al. Plant Biology 2010

What can we say about it ?

Folds

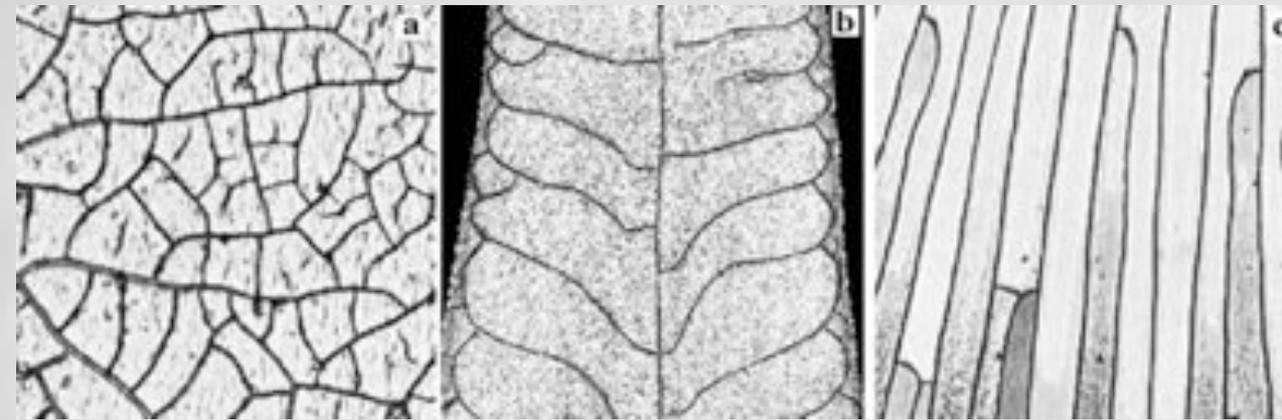


Schizophragma hydrangeoides

reticulated

Visual analogy between venation & crack patterns

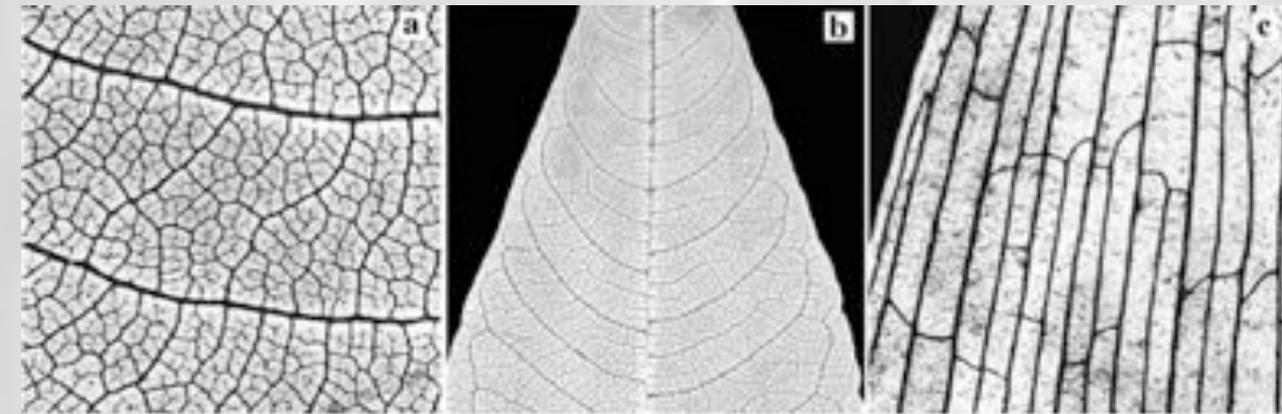
Crack Patterns



Pattern controlled
by a tensorial
field,

Couder et al., 2002

Venation Patterns

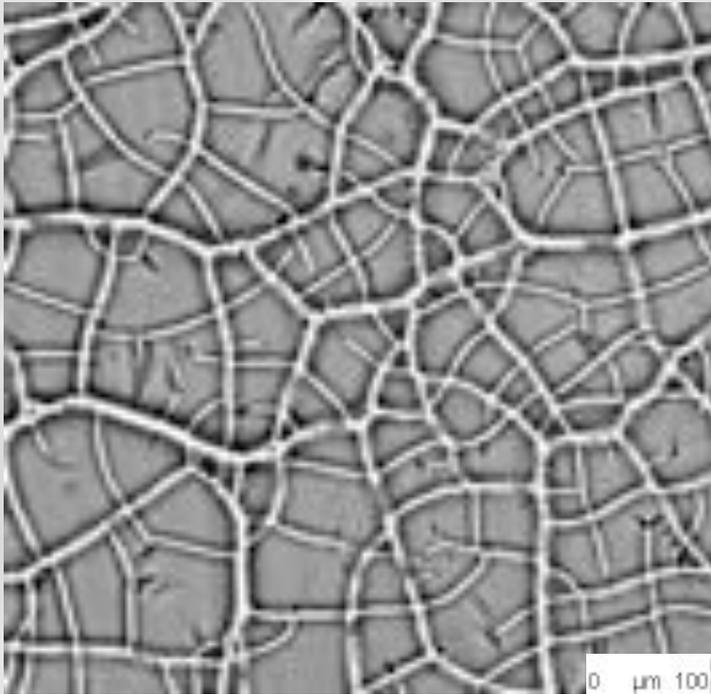


while [Auxin] is a
scalar field.

Is this field represented by mechanical stress ?

Visual analogy between Fold & Crack patterns

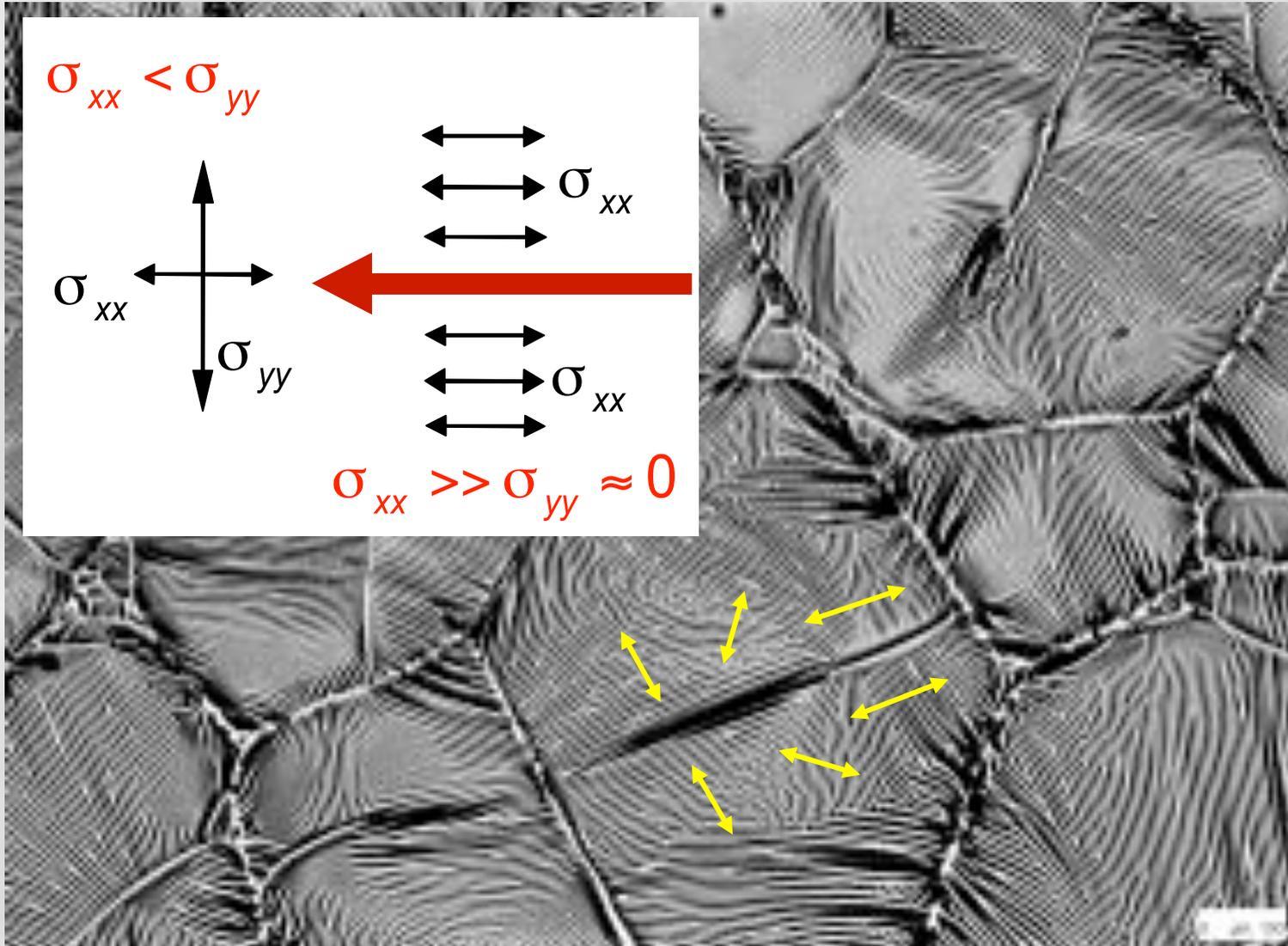
In compression



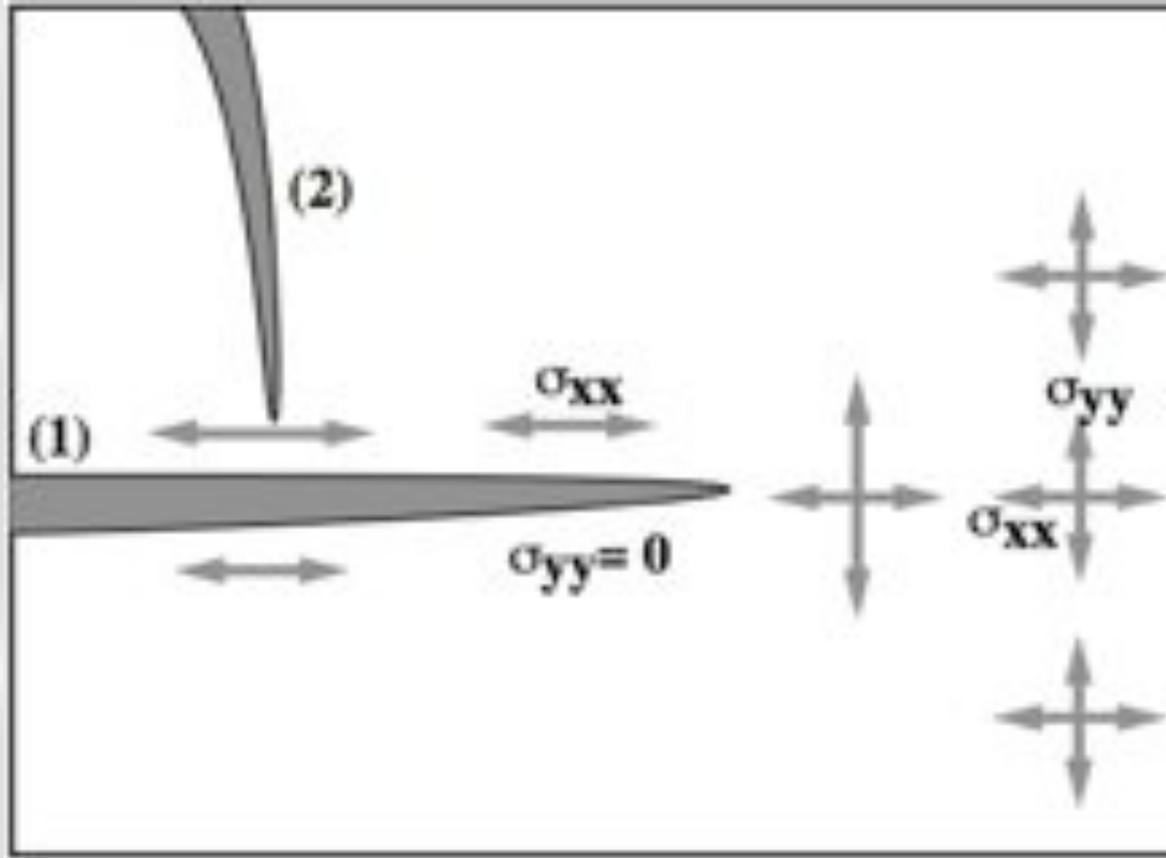
In tension



Reorganization of stress distribution: Visible thanks to the wrinkles !



General mechanism to produce a pattern

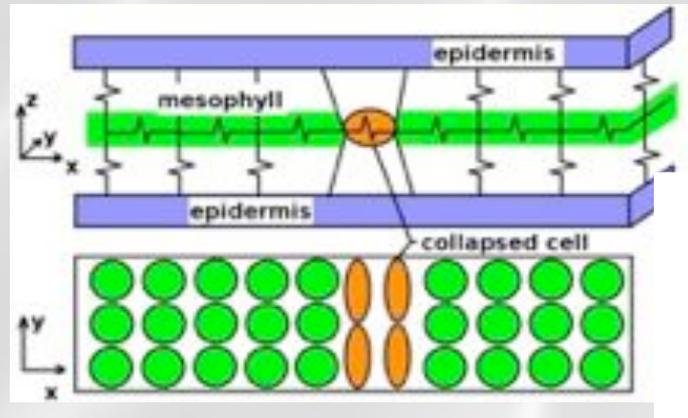
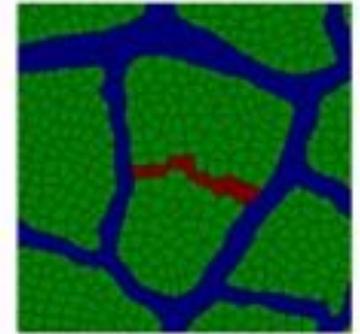
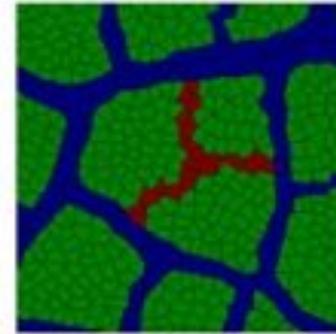
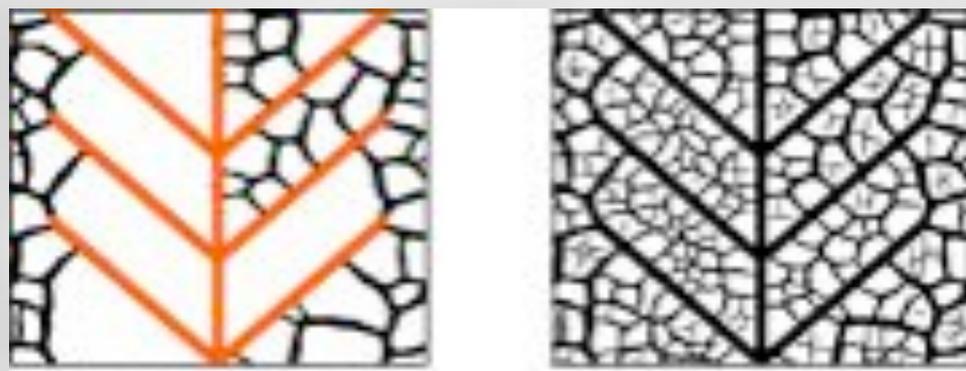


The **hierarchical division of space** is fundamentally linked to the **tensorial nature of the stress field** and the **local and anisotropic release of stress**

Venation in silico: stress release mechanism

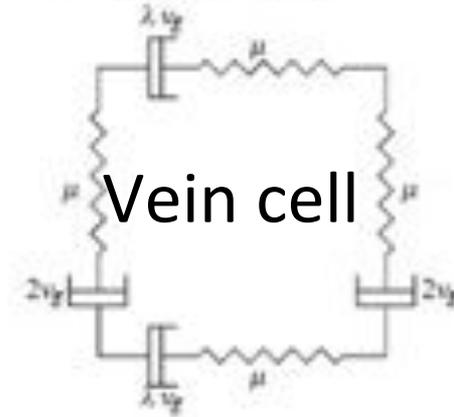
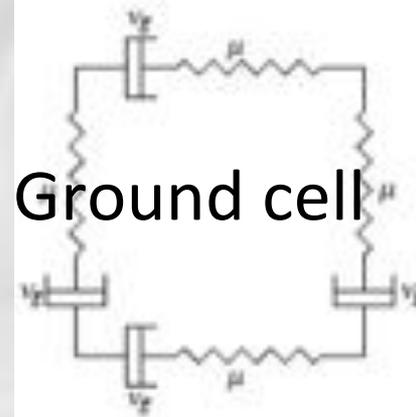
Laguna *et al*, *PLoS Comput. Biol.* (2008)

Corson *et al*, *J. Theor Biol* (2009)



Ground cell

Vein cell

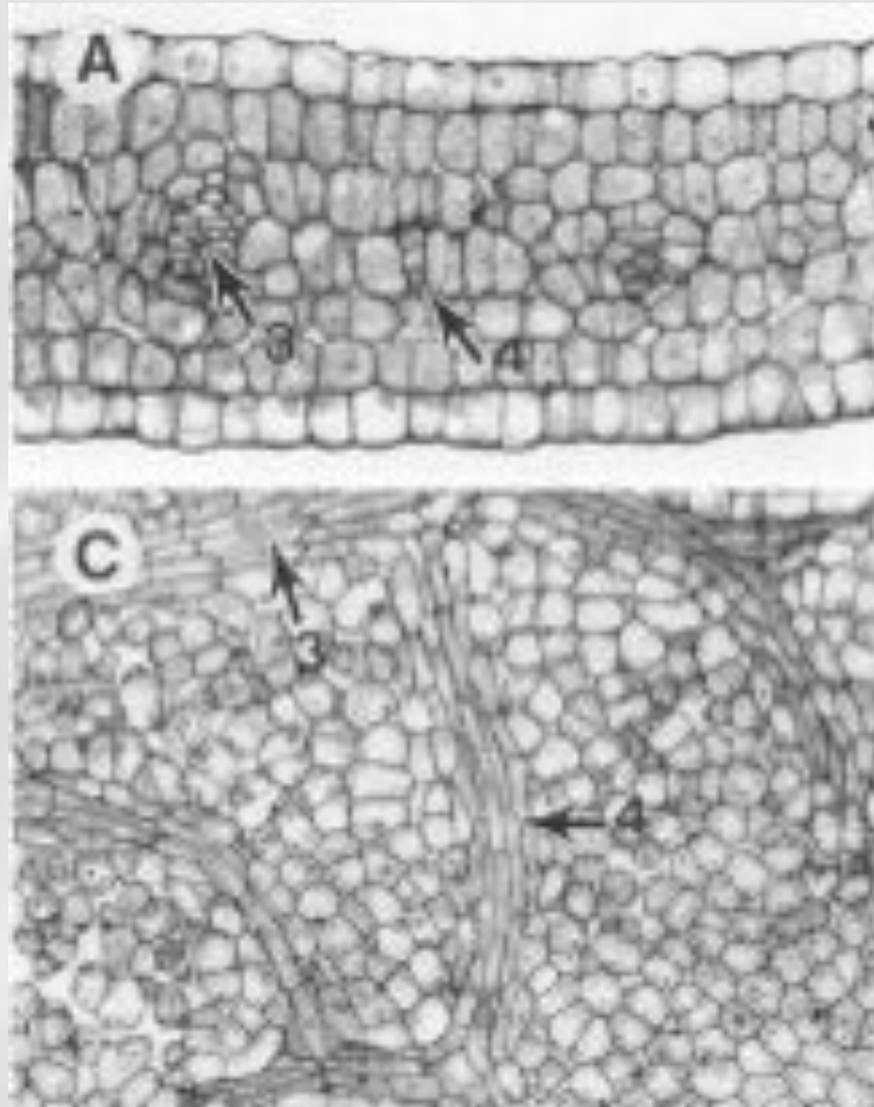


Irreversible collapsible state for cells

Different mechanical response between veins and ground cells

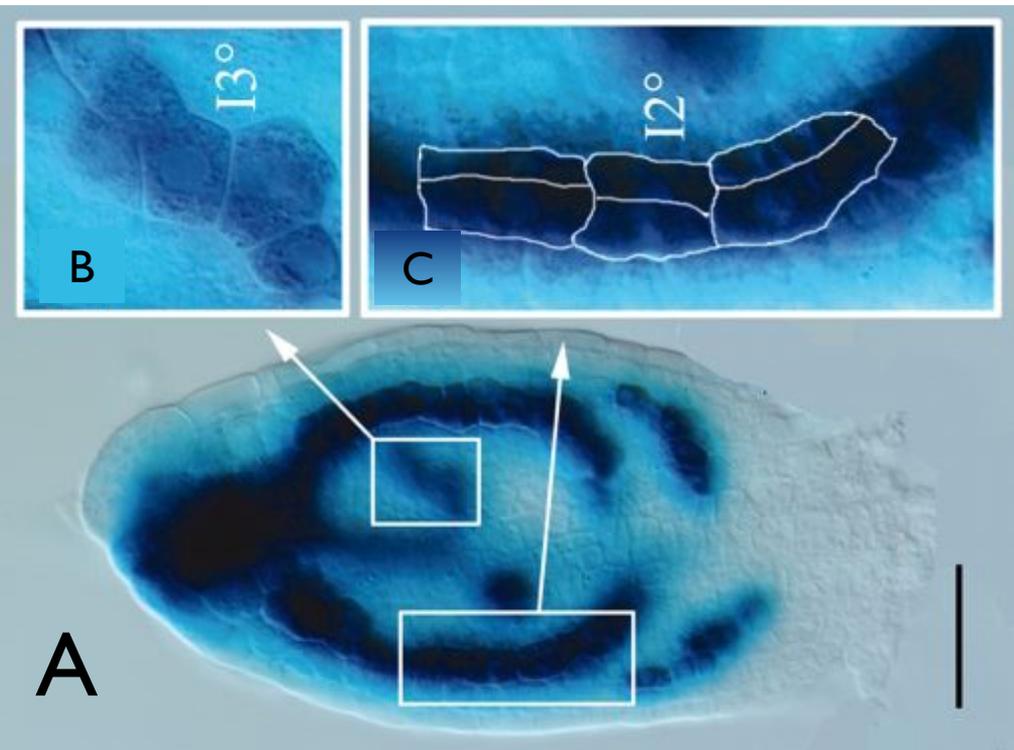
Produce a reticulated pattern. Collapse in leaves ?

Procambial cells are indeed collapsed



Nelson *et al*, *Plant Cell* (1997)

But an analogy is not an homology !



Other fields to take into consideration !

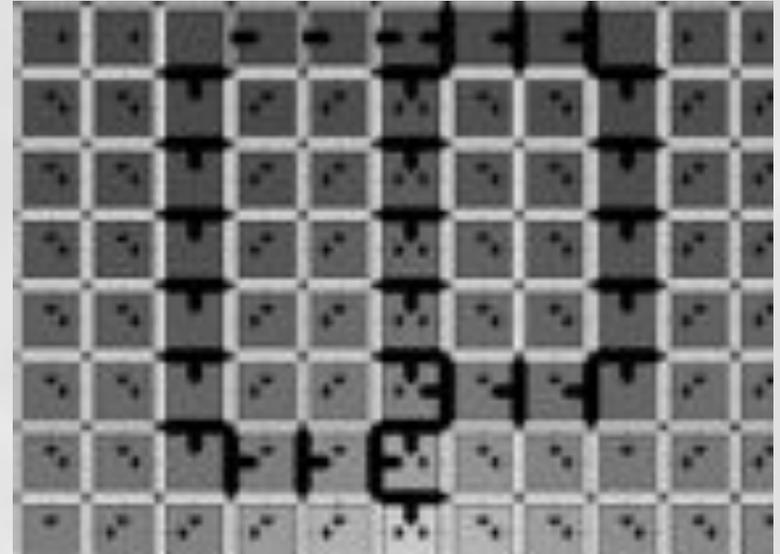
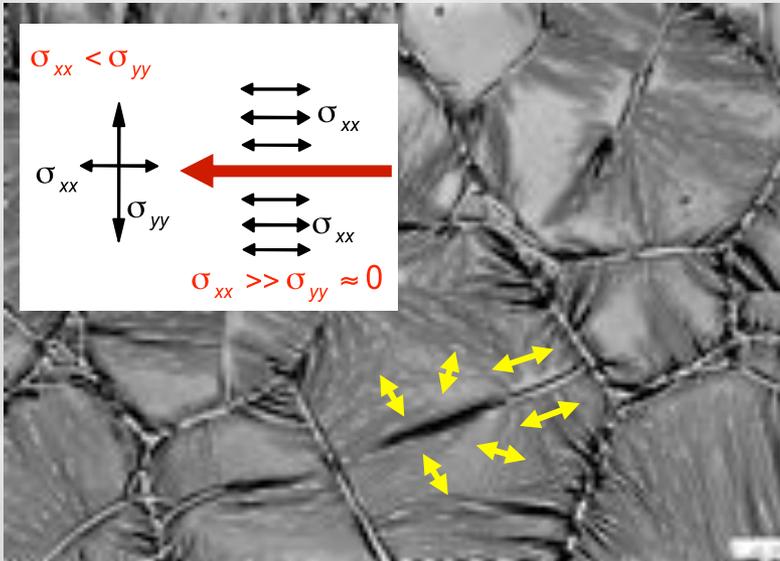
[Auxin] link to elasticity ?

If the stress field is not important Why such resemblance ?

Stress ?

vs

Polar transport ?



Equivalent mathematically ?

What tensor ?

stress tensor

$$\sigma_{ij}$$

permeability tensor

$$\kappa_{ij}$$

A vein modifies locally and anisotropically
the permeability tensor to Auxin

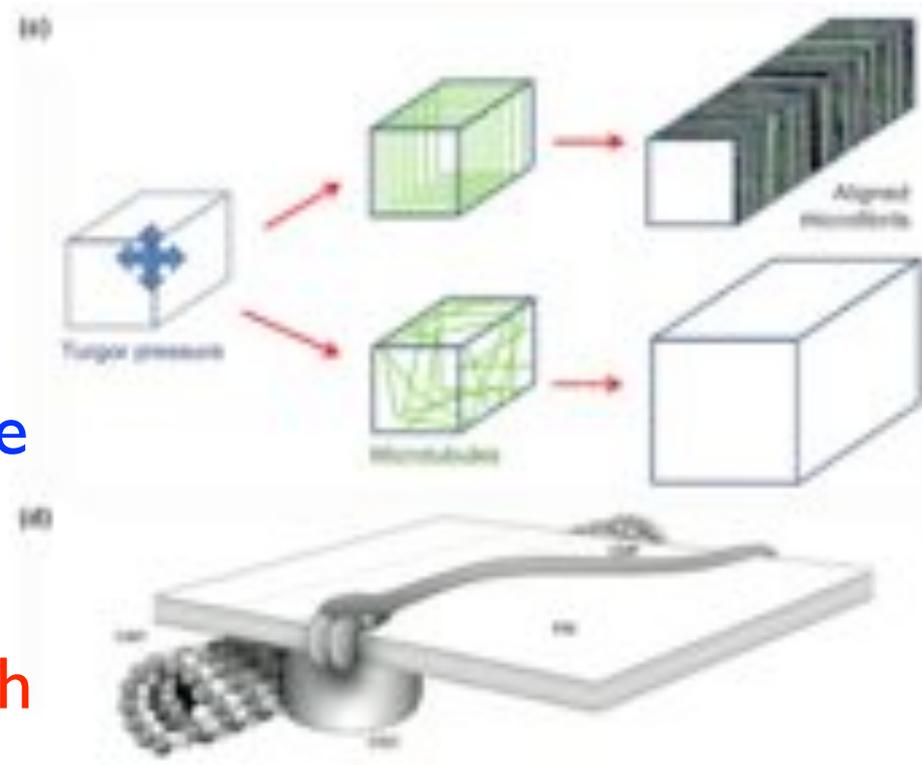
Link between transport and stress ?

Role of Auxin and its transport ?

Auxin controls local growth

Cells have microtubules
They align with stress or before
division

Anisotropic response to growth
and to permeability ?



Hamant and Traas, 2010

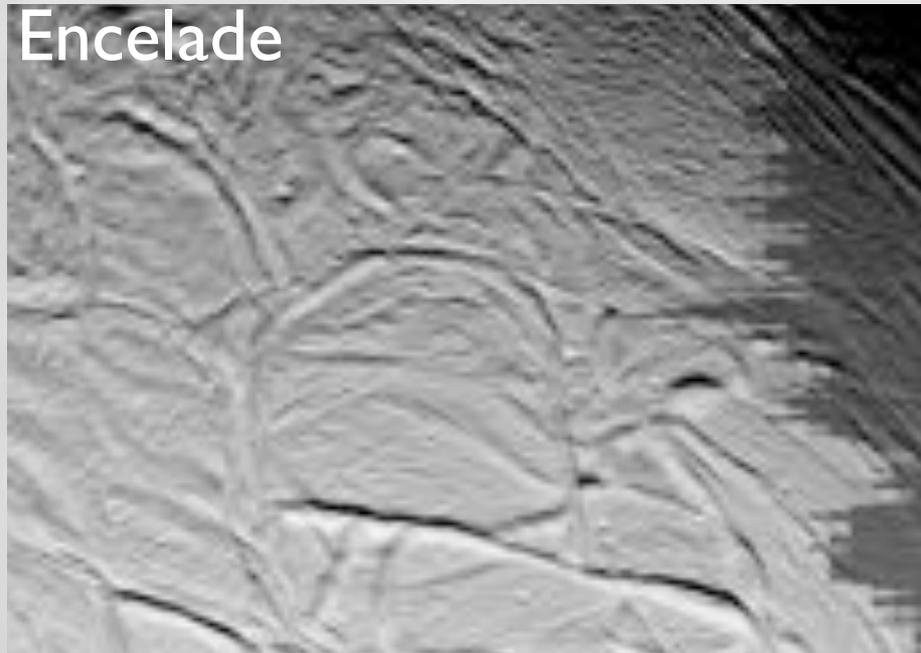


What do you think ?

Canalization hypothesis realized in a foam ?



Delamination network



[Source](#) : NASA/JPL/Space Science Institute

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